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M a t h e m a t i c s

NONLOCAL PROBLEM FOR A MIXED TYPE DIFFERENTIAL EQUATION IN RECTANGULAR DOMAIN

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In the article the questions of solvability and construction of the solution of nonlocal mixed value problem for a homogeneous mixed type differential equation are considered. The spectral method based on the separation of variables is used. A criterion for a single-valued solvability of the considered problem is installed. Under this criterion the single-valued solvability of the problem is proved. The existence of problem solutions in the case of uniqueness failure is studied, also.

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Problem Statement. The partial differential equations of third and fourth order are important in the physical applications [1–5]. Boussinesq type differential equations have many applications in mathematical physics [6].

Various direct and inverse problems for partial differential equations of third and fourth order are studied in a large number of works (see, for examples, [7–13]).

When the boundary of the physical process is not available for measurement, as an additional information for the unique solvability of the problem, it can be used the nonlocal conditions in the integral form [14].

The problems, where the type of differential equation is changed in the considering domain, have important applications [15–17]. The mixed type differential equations have been studied by many authors, in particular in [18–25].

In the present paper a single-valued solvability of nonlocal problem for a mixed type differential equation with an integral condition is established. So, in a rectangular domain $\Omega = \{(t,x)| -\alpha < t < \beta;\ 0 < x < 1\}$, where α and β are given positive real numbers, consider the following mixed type equation

$$\Im U \equiv \begin{cases} U_t - U_{txx} - U_{xx} = 0, & t > 0, \\ U_{tt} - U_{ttxx} - U_{xx} = 0, & t < 0. \end{cases} \tag{1}$$

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The first equation of (1) is from the class of Boussinesq type equation [6], as the second one is from class of pseudoparabolic type equations.

Problem. Find a function $U(t,x)$ in the domain Ω, satisfying the following conditions:

$$U(t,x)\in C\big(\overline{\Omega}\big)\cap C^1\big(\Omega\cup\{x=0\}\cup\{x=1\}\big)\cap C^2\big(\Omega_-\big)\cap C^{1,2}_{t,x}\big(\Omega_+\cup\{t=\beta\}\big),\tag{2}$$

$$\Im U(t,x)\equiv 0,\ (t,x)\in\Omega_-\cup\Omega_+\cup\{t=\beta\},\tag{3}$$

$$U(t,0)=U(t,1),\ U_x(t,0)=U_x(t,1),\ \ -\alpha\le t\le\beta,\tag{4}$$

$$\int\limits_{-\alpha}^{0}U(t,x)tdt=\psi(x),\ 0\le x\le 1,\tag{5}$$

where $\psi(x)$ is a given sufficiently smooth function, $\psi(0)=\psi(1)$, $\psi'(0)=\psi'(1)$, $\Omega_-=\{(t,x)|-\alpha<t<0,\ 0<x<1\}$, $\Omega_+=\{(t,x)|0<t<\beta,\ 0<x<1\}$.

Particular Solutions. The nontrivial partial solutions of the Eq. (1) in the domain Ω we find in form $U(t,x)=T(t)\cdot X(x)$. In accordance to Eq. (1)

$$\begin{cases} T'(t)\cdot X(x)-T'(t)\cdot X''(x)=T(t)\cdot X''(x),\ t>0,\\ T''(t)\cdot X(x)-T''(t)\cdot X''(x)=T(t)\cdot X''(x),\ t<0.\end{cases}$$

Dividing by $T(t)\cdot X(x)$ and then setting $\dfrac{X''(x)}{X(x)}=-\mu^2$, we get:

$$\frac{T'(t)}{T(t)}-\frac{X''(x)}{X(x)}\cdot\frac{T'(t)}{T(t)}=-\mu^2\ \text{as}\ t>0,\ \frac{T''(t)}{T(t)}-\frac{X''(x)}{X(x)}\cdot\frac{T''(t)}{T(t)}=-\mu^2\ \text{as}\ t<0,$$

where μ^2 is the separation permanent, $0<\mu$.

Hence, taking into account the boundary conditions (4), we derive

$$X''(x)+\mu^2X(x)=0,\ 0<x<1,\quad X(0)=X(0),\ X'(0)=X'(1),\tag{6}$$

$$T'(t)+\lambda^2T(t)=0,\ 0<t<\beta,\ \ T''(t)+\lambda^2T(t)=0,\ -\alpha<t<0,\tag{7}$$

where $\lambda^2=\mu^2/(1+\mu^2)$.

The spectral problem (6) has the solution

$$X_0(x)=1,\ X_n(x)=\big\{\cos\mu_nx;\ \sin\mu_nx\big\},\ \mu_n=2\pi n,\ n=1,2,\dots\tag{8}$$

The general solution of differential Eqs. (7) has the form with a_n,b_n,c_n arbitrary constants:

$$T_n(t)=\begin{cases} c_ne^{-\lambda_n^2}t,\ t>0,\\ a_n\cos\lambda_nt+b_n\sin\lambda_nt,\ t<0.\end{cases}\tag{9}$$

The solutions $U_n(t,x)=T_n(t)\cdot X_n(x)$ must satisfy the conditions (2), so, constants a_n, b_n and c_n will be chosen to satisfy the following conditions:

$$T_n(0+0)=T_n(0-0),\ T_n'(0+0)=T_n'(0-0).\tag{10}$$

From (9), taking into account the conditions (10), we obtain $a_n=c_n$ and $b_n=-\lambda_nc_n$. Then the functions (9) take the form

$$T_n(t)=\begin{cases} c_ne^{-\lambda_n^2}t,\ t>0,\\ c_n\cos\lambda_nt-\lambda_nc_n\sin\lambda_nt,\ t<0.\end{cases}\tag{11}$$

Let present solution of the problem (2)–(5) in the domain Ω according to the Fourier method and taking into account (8) in the following form:

$$U(t,x)=\frac{\vartheta_0(t)}{2}+\sum_{n=1}^{\infty}\Big[\vartheta_n(t)\cdot\cos\mu_n x+u_n(t)\sin\mu_n x\Big],$$

where the Fourier coefficients

$$u_n(t)=2\int_0^1 U(t,x)\sin\mu_n x dx,\ n=1,2,\dots, \tag{12}$$

$$\vartheta_n(t)=2\int_0^1 U(t,x)\cos\mu_n x dx,\ n=0,1,2,\dots \tag{13}$$

Determination of the Fourier Coefficients. We show that functions (12), (13) satisfy the Eq. (7) in the corresponding intervals as well as the condition (10). Differentiation of Eqs. (12), (13) with respect to t (once in the case $t>0$, and twice as $t<0$), taking into account Eq. (1), implies

$$u_n'(t)=2\int_0^1 U_t\sin\mu_n x dx=2\int_0^1\Big(U_{txx}+U_{xx}\Big)\sin\mu_n x dx, \tag{14}$$

$$u_n''(t)=2\int_0^1 U_{tt}\sin\mu_n x dx=2\int_0^1\Big(U_{ttxx}+U_{xx}\Big)\sin\mu_n x dx, \tag{15}$$

$$\vartheta_n'(t)=2\int_0^1 U_t\cos\mu_n x dx=2\int_0^1\Big(U_{txx}+U_{xx}\Big)\cos\mu_n x dx, \tag{16}$$

$$\vartheta_n''(t)=2\int_0^1 U_{tt}\cos\mu_n x dx=2\int_0^1\Big(U_{ttxx}+U_{xx}\Big)\cos\mu_n x dx. \tag{17}$$

Integrating by parts twice in the integrals (14)–(17), taking into account (4), the following equations are derived (as before $\lambda_n^2=\mu_n^2/(1+\mu_n^2)$):

$$u_n'(t)+\lambda_n^2 u_n(t)=0,\ t>0, \tag{18}$$

$$u_n''(t)+\lambda_n^2 u_n(t)=0,\ t<0, \tag{19}$$

$$\vartheta_n'(t)+\lambda_n^2\vartheta_n(t)=0,\ t>0, \tag{20}$$

$$\vartheta_n''(t)+\lambda_n^2\vartheta_n(t)=0,\ t<0. \tag{21}$$

The differential Eqs. (18), (19) and (20), (21) for $\lambda=\lambda_n$ coincide with the left and right differential equations from (7) respectively. Further, taking the conditions (2), from (12) and (13), we get

$$u_n(0+0)=2\int_0^1 U(0+0,x)\sin\mu_n x dx=2\int_0^1 U(0-0,x)\sin\mu_n x dx=u_n(0-0), \tag{22}$$

$$\vartheta_n(0+0) = 2\int_0^1 U(0+0,x)\cos\mu_n x dx = 2\int_0^1 U(0-0,x)\cos\mu_n x dx = \vartheta_n(0-0). \quad (23)$$

Differentiating (12), (13) with respect to t by virtue of conditions (2), we obtain:

$$u_n'(0+0) = 2\int_0^1 U_t(0+0,x)\sin\mu_n x dx = 2\int_0^1 U_t(0-0,x)\sin\mu_n x dx = u_n'(0-0), \quad (24)$$

$$\vartheta_n'(0+0) = 2\int_0^1 U_t(0+0,x)\cos\mu_n x dx = 2\int_0^1 U_t(0-0,x)\cos\mu_n x dx = \vartheta_n'(0-0). \quad (25)$$

Eqs. (22), (23) and (24), (25) coincide with conditions (10). Then for problems (18)–(25) analogously to (11) we obtain

$$u_n(t) = \begin{cases} c_n e^{-\lambda_n^2} t, \ t > 0, \\ c_n \cos\lambda_n t - \lambda_n c_n \sin\lambda_n t, \ t < 0, \end{cases} \quad (26)$$

$$\vartheta_n(t) = \begin{cases} \tilde{c}_n e^{-\lambda_n^2} t, \ t > 0, \\ \tilde{c}_n \cos\lambda_n t - \lambda_n \tilde{c}_n \sin\lambda_n t, \ t < 0. \end{cases} \quad (27)$$

To find constants c_n and $\tilde{c}_n$, integral condition (5) and Eqs. (12), (13) are used:

$$\int_{-\alpha}^0 u_n(t) t dt = 2\int_0^1\int_{-\alpha}^0 U(t,x) t dt \sin\mu_n x dx = 2\int_0^1 \psi(x)\sin\mu_n x dx = \psi_n, \quad (28)$$

$$\int_{-\alpha}^0 \vartheta_n(t) t dt = 2\int_0^1\int_{-\alpha}^0 U(t,x) t dt \cos\mu_n x dx = 2\int_0^1 \psi(x)\cos\mu_n x dx = \tilde{\psi}_n. \quad (29)$$

Then, since $t < 0$, from (26) and (28) we obtain

$$\psi_n = \int_{-\alpha}^0 u_n(t) t dt = c_n \int_{-\alpha}^0 (\cos\lambda_n t - \lambda_n \sin\lambda_n t)\, t dt =$$

$$= c_n \left[\frac{1}{\lambda_n^2} - \left(\frac{1}{\lambda_n^2} - \alpha \right) \cos\lambda_n\alpha - \frac{1+\alpha}{\lambda_n} \sin\lambda_n\alpha \right],$$

i.e.

$$c_n \Delta_n(\alpha) = \psi_n, \quad (30)$$

where $\Delta_n(\alpha) = \frac{1}{\lambda_n^2} - \left(\frac{1}{\lambda_n^2} - \alpha \right) \cos\lambda_n\alpha - \frac{1+\alpha}{\lambda_n} \sin\lambda_n\alpha.$

Analogously, from (27) and (29) we obtain

$$\tilde{c}_n \Delta_n(\alpha) = \tilde{\psi}_n. \quad (31)$$

Suppose, that

$$\Delta_n(\alpha) \neq 0. \tag{32}$$

Taking into account(32), from (30), (31) we obtain $c_n = \dfrac{\psi_n}{\Delta_n(\alpha)}$, $\tilde{c}_n = \dfrac{\tilde{\psi}_n}{\Delta_n(\alpha)}$. Substituting c_n and $\tilde{c}_n$ into (26) and (27), we derive

$$u_n(t) = \begin{cases} A_n \psi_n e^{-\lambda_n^2} t, \ t > 0, \\ A_n \Big(\cos \lambda_n t - \lambda_n c_n \sin \lambda_n t \Big) \cdot \psi_n, \ t < 0, \end{cases} \tag{33}$$

$$\vartheta_n(t) = \begin{cases} A_n \tilde{\psi}_n e^{-\lambda_n^2} t, \ t > 0, \\ A_n \Big(\cos \lambda_n t - \lambda_n c_n \sin \lambda_n t \Big) \cdot \tilde{\psi}_n, \ t < 0, \end{cases} \tag{34}$$

where $A_n = 1/\Delta_n(\alpha)$.

Supposing $\psi(x) \equiv 0$, and $\psi_n = \tilde{\psi}_n \equiv 0$, from (12), (13) and (33), (34) we get

$$\int_0^l U(t,x) \cdot \sin \mu_n x dx = 0, n = 1,2,\ldots, \quad \int_0^l U(t,x) \cdot \cos \mu_n x dx = 0, n = 0,1,2,\ldots$$

Hence, by the completeness of the system of eigenfunctions $\{1, \cos \mu_n x, \sin \mu_n x\}$ in the space $L_2[0,1]$, we deduce that $U(t,x) \equiv 0$ for all $x \in [0,1]$ and $t \in [-\alpha, \beta]$.

We consider the case of failure of the condition (32). Let $\Delta_n(\alpha) = 0$ for some α and $n = m$. Then homogeneous problem (2)–(5) as $\psi(x) \equiv 0$ has a nontrivial solution

$$U_m(t,x) = T_m(t) \cdot X_m(x), \tag{35}$$

where $X_m(x)$: $\{1, \cos \mu_n x, \sin \mu_n x\}$,

$$T_m(t) = \begin{cases} e^{-\lambda_m^2} t, \ t > 0, \\ \cos \lambda_m t - \lambda_m \sin \lambda_m t, \ t < 0. \end{cases}$$

The condition $\Delta_n(\alpha) = 0$ is equivalent to the equality

$$\left(1 - \lambda_n^2 \alpha\right) \cos \lambda_n \alpha + \lambda_n (\alpha + 1) \sin \lambda_n \alpha = 1, \tag{36}$$

where $\lambda_n = \sqrt{\dfrac{\mu_n^2}{1 + \mu_n^2}}$, $\mu_n = 2\pi n$. Here $0 < \lambda_n < 1$, $\lambda_n \to 1$ as $n \to \infty$.

From Eq. (36) the following is obtained:

$$\sin\left(\lambda_n \alpha + \varphi_n\right) = \frac{1}{\sqrt{\lambda_n^2 \gamma + \left(1 - \lambda_n^2 \alpha\right)^2}}, \tag{37}$$

where $\gamma = (\alpha + 1)^2$, $\varphi_n = \arcsin \frac{\lambda_n (\alpha+1)}{\sqrt{\lambda_n^2 \gamma + \left(1 - \lambda_n^2 \alpha\right)^2}}$.

Observe that the condition $0 < \frac{1}{\sqrt{\lambda_n^2\gamma + \left(1-\lambda_n^2\alpha\right)^2}} < 1$ is satisfied for any $0 < \alpha$ and n. Indeed, from $\sqrt{\lambda_n^2\gamma + \left(1-\lambda_n^2\alpha\right)^2} > 1$ we get $(\alpha+1)^2\lambda_n^2 + \lambda_n^4\alpha^2 > 0$.

Hence, the Eq. (37) has the following solution:

$$\alpha_k = \frac{\theta_n - \varphi_n}{\lambda_n} + \frac{\pi k}{\lambda_n},\ k = 1,2,3,\ldots,$$

where $\theta_n = (-1)^k \arcsin \frac{1}{\sqrt{\lambda_n^2\gamma + \left(1-\lambda_n^2\alpha\right)^2}}$.

Recall other values of α, for which condition (32) holds, regular values.

We show that there exists a constant $C_0 > 0$ such that for a sufficiently large n the estimate holds

$$\inf_n \left|\Delta_n(\alpha)\right| \geq C_0. \tag{38}$$

Suppose there exists a constant $C_0 > 0$ such that for a sufficiently large n we have the estimate (38). Then from (32), taking into account $0 < \lambda_n < 1$ and $\lambda_n \to 1$ as $n \to \infty$, we obtain

$$\left|1 - \sqrt{2(1+\alpha^2)}\sin(\alpha+\varphi)\right| \geq C_0,\ \text{ where }\ \varphi = \arcsin\frac{(\alpha+1)}{\sqrt{2(1+\alpha^2)}}.$$

The last inequality holds if $\left|1 - \sqrt{2(1+\alpha^2)}\sin(\alpha+\varphi)\right| > 0.$

This inequality is equivalent to the aggregate of the following two inequalities:

$$\sin(\alpha+\varphi) < \frac{1}{\sqrt{2(1+\alpha^2)}},\ \ \sin(\alpha+\varphi) > \frac{1}{\sqrt{2(1+\alpha^2)}}. \tag{39}$$

Since $\sqrt{2(1+\alpha^2)} > 1$, the trigonometric inequalities (39) have a solution. Consequently, the estimate (38) is true for a sufficiently large n and for any $0 < \alpha$, satisfying to one of two inequalities (39).

Existence of Solution. For the regular values of α the formulas (33) and (34) are satisfied. Therefore, under (32) and (38), taking into account the particular solutions (8), (33) and (34), the solution of the problem (2)–(5) in the domain Ω can be represent by series

$$U(t,x) = \frac{\vartheta_0(t)}{2} + \sum_{n=1}^{\infty}\left[\vartheta_n(t)\cos\mu_n x + u_n(t)\sin\mu_n x\right]. \tag{40}$$

We show that the sum $U(t,x)$ of the series (40) under certain conditions on the function $\psi(x)$ satisfies to the conditions (2).

It is easy to check, for a sufficiently large n there the following estimates hold:

$$\left|u_n(t)\right| \leq C\left|\psi_n\right|,\ \ \left|\vartheta_n(t)\right| \leq C\left|\tilde{\psi}_n\right|, \tag{41}$$

$$\left|u_n'(t)\right| \leq C\left|\psi_n\right|,\ \ \left|\vartheta_n'(t)\right| \leq C\left|\tilde{\psi}_n\right|, \tag{42}$$

$$\left|u_n''(t)\right| \leq C\left|\psi_n\right|, \ \left|\vartheta_n''(t)\right| \leq C\left|\tilde{\psi}_n\right|, \tag{43}$$

where $0 < C =$const.

Indeed, according to (33), (34) and taking into account (38), we find

$$\left|u_n(t)\right| = \begin{cases} \dfrac{1}{C_0}\left|\psi_n\right|, \ t>0, \\ \dfrac{2}{C_0}\left|\psi_n\right|, \ t<0, \end{cases} \quad \left|\vartheta_n(t)\right| = \begin{cases} \dfrac{1}{C_0}\left|\tilde{\psi}_n\right|, \ t>0, \\ \dfrac{2}{C_0}\left|\tilde{\psi}_n\right|, \ t<0. \end{cases}$$

Hence, we get the estimate (41), if we set $C = 2/C_0$.

After the differentiating (33) and (34), we obtain

$$\left|u_n'(t)\right| = \begin{cases} \dfrac{1}{C_0}\left|\psi_n\right|, t>0, \\ \dfrac{2}{C_0}\left|\psi_n\right|, t<0, \end{cases} \quad \left|\vartheta_n'(t)\right| = \begin{cases} \dfrac{1}{C_0}\left|\tilde{\psi}_n\right|, t>0, \\ \dfrac{2}{C_0}\left|\tilde{\psi}_n\right|, t<0, \end{cases}$$

and so the estimate (42). Here we take $C = 2/C_0$.

Differentiating (33) and (34) twice for $t<0$, we obtain

$$\left|u_n''(t)\right| = \frac{2}{C_0}\left|\psi_n\right|, \ \left|\vartheta_n''(t)\right| = \frac{2}{C_0}\left|\tilde{\psi}_n\right|,$$

and hence the estimate (43) for $C = 2/C_0$.

Since the function $\psi(x) \in C^3[0,1]$ has piecewise continuous derivative of fourth order in the segment $[0,1]$ and $\psi(0)=\psi(1)$, $\psi'(0)=\psi'(1)$, $\psi''(0)=\psi''(1)$, $\psi'''(0)=\psi'''(1)$, then following estimates hold:

$$\psi_n = -\left(\frac{1}{\pi}\right)^4 \frac{p_n}{n^4}, \ \sum_{n=1}^{\infty} p_n^2 \leq 4\int_0^1 \left[\psi^{IV}(x)\right]^2 dx,$$

$$\tilde{\psi}_n = -\left(\frac{1}{\pi}\right)^4 \frac{q_n}{n^4}, \ \sum_{n=1}^{\infty} q_n^2 \leq 4\int_0^1 \left[\psi^{IV}(x)\right]^2 dx.$$

Using these estimates, it is not difficult to see that the series (40) and the series of first order terms of the series converge uniformly in the domain $\overline{\Omega}$.

Let $\Delta_n(\alpha)=0$ for some α, $n=k_1,\ldots,k_s$, where $1 \leq k_1 < k_2 < \cdots < k_s$ and s is a fixed natural number. Then for the solvability of Eqs. (30) and (31) it is necessary and sufficient the orthogonality conditions

$$\psi_n = 2\int_0^1 \psi(x)\sin 2\pi nx\,dx = 0, \ \ n=k_1,\ldots,k_s, \tag{44}$$

$$\tilde{\psi}_n = 2\int_0^1 \psi(x)\cos 2\pi nx\,dx = 0, \ \ n=k_1,\ldots,k_s. \tag{45}$$

In this case the solution of the problem (2)–(5) is defined as the sum of the series

$$U(t,x) = \frac{\vartheta_0(t)}{2} + \left(\sum_{n=1}^{k_1-1} + \sum_{n=k_1+1}^{k_2-1} + \cdots + \sum_{n=k_s+1}^{\infty} \right) u_n(t) \sin \mu_n x + \\ + \left(\sum_{n=1}^{k_1-1} + \sum_{n=k_1+1}^{k_2-1} + \cdots + \sum_{n=k_s+1}^{\infty} \right) \vartheta_n(t) \cos \mu_n x + \sum_m C_m U_m(t,x), \tag{46}$$

where m takes values $k, k_1, \ldots, k_s$, C_m are arbitrary constants and functions $U_m(t,x)$ are defined in (35).

Thus, the following Theorem is proved.

Theorem. Let the function $\psi(x) \in C^3[0,1]$ has piecewise continuous derivative of fourth order in the segment $[0,1]$ and $\psi(0) = \psi(1)$, $\psi'(0) = \psi'(1)$, $\psi''(0) = \psi''(1)$, $\psi'''(0) = \psi'''(1)$. Then the problem (2)–(5) in the domain Ω is uniquely solvable, whenever conditions (32), (38) are satisfied. This solution is determined by the series (40). Let $\Delta_n(\alpha) = 0$ for some α, $n = k_1, \ldots, k_s$ and the condition (38) is satisfied. Then the problem (2)–(5) is solvable if the orthogonality conditions (44) and (45) hold. This solution is defined by the series (46).

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