PROCEEDINGS OF THE YEREVAN STATE UNIVERSITY

Physical and Mathematical Sciences 2016, № 3, p. 11–14

Physics

ON A POSSIBLE INFLATIONARY MODEL OF THE EARLY UNIVERSE

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We consider the inflationary regime of the expansion of the Universe within the framework of the modified Jordan theory. In this model, a specific cosmological scalar is an analog of the Einstein cosmological constant Λ, which is connected with the Hubble parameter by the power-law $\varphi(\Phi) = \alpha H^4$. As a result, a stage of hybrid inflation is obtained, at the end of which the acceleration parameter becomes zero.

***Keywords*:** inflation, scalar-tensor theory of gravitation.

Inflation with a Minimally Coupled Scalar Field. It is well known that the study of vacuum phenomena at quantum level has led to the conclusion of their possible responsibility for the cosmological constant Λ. Models of the early Universe, under the assumption $\Lambda\tilde{~}H^n$, have been discussed in [1, 2].

In the presence of the cosmological scalar $\varphi(\Phi)$, the model with a minimally coupled scalar field is described by the action functional [3]

$$S = \int \sqrt{-g}\left[-\frac{y_0}{16\pi}\left(R + 2\varphi(\Phi)\right) + \frac{1}{2}g^{\alpha\beta}\Phi_\alpha\Phi_\Phi\right]d^4x, \tag{1}$$

were $y_0 = 2(2+\zeta)/G(3+2\zeta)$. For the standard Friedmann–Robertson–Walker (FRW) line element

$$ds^2 = dt^2 - a^2(t)\left(dr^2 + r^2 d\Omega^2\right) \tag{2}$$

with flat space, the field equations in the absence of the ordinary matter ($\varepsilon = 0$, $P = 0$) take the form

$$\ddot{\Phi} + 3H\dot{\Phi} + V'(\Phi) = 0, \tag{3}$$

$$3H^2 = \frac{8\pi}{y_0}\left(\frac{\dot{\Phi}^2}{2} + V(\Phi)\right), \tag{4}$$

$$2\dot{H} + 3H^2 = -\frac{8\pi}{y_0}\left(\frac{\dot{\Phi}^2}{2} - V(\Phi)\right). \tag{5}$$

Here we have introduced the notations $V(\Phi) = y_0\varphi(\Phi)/8\pi$, $H = \dot{a}/a$, and the dot and prime stand for the derivatives with respect to time and to the scalar field Φ respectively. By taking into account that H is a function of Φ, from (4) and (5) the following relations are obtained

$$2\dot{H} = -\frac{8\pi}{y_0}\dot{\Phi}^2,\ 2H' = -\frac{8\pi}{y_0}\dot{\Phi}. \tag{6}$$

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Combining this with (4) one gets

$$\dot{H} = \varphi(\Phi) - 3H^2, \tag{7}$$

$$2H'^2 = \frac{8\pi}{y_0}\left[3H^2 - \varphi(\Phi)\right]. \tag{8}$$

For a given function $\varphi(\Phi)$, these relations provide the dependence of the Hubble function on time and on Φ.

Hubble Function. Here we consider the model with $V = \alpha H^4$ $(\alpha > 0)$. The dependence of the Hubble function on the scalar Φ is directly obtained from (8). Introducing the notation $H_0^2 = 3/\alpha$ and assuming that $H > H_0$, after the simple integration we get

$$H = \frac{H_0}{\cosh\left(2\sqrt{3\pi/y_0}\Phi\right)}. \tag{9}$$

The corresponding potential energy has the form

$$V(\Phi) = \frac{3y_0}{8\pi}H_0^2\left(\frac{H}{H_0}\right)^4. \tag{10}$$

In order to find the time dependence of the Hubble function, we write the Eq. (7) in the form

$$\frac{d}{dt}\left(\frac{H}{H_0}\right) = 3H_0\left[\left(\frac{H}{H_0}\right)^4 - \left(\frac{H}{H_0}\right)^2\right]. \tag{11}$$

The corresponding is presented as

$$3H_0 t = \frac{H_0}{H} + \frac{1}{2}\ln\left(\frac{H_0 - H}{H_0 + H}\right), \tag{12}$$

where, as before, we have assumed that $H_0 > H$. For the integration constant we have taken $t_0 = 1/(3H_0)$. For the acceleration parameter $q = \dot{a}a/\dot{a}^2 = \dot{H}/H + 1$ on gets the expression

$$q = 3(H/H_0)^2 - 2, \tag{13}$$

which vanishes at $H = H_c = \sqrt{2/3}H_0$.

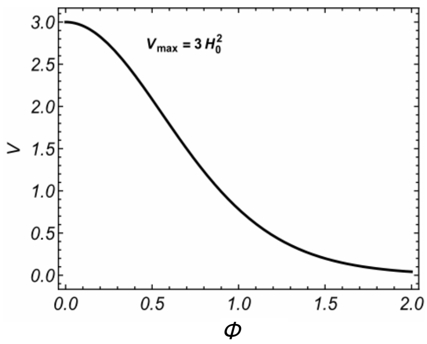

Fig. 1. The scalar potential for the model under consideration.

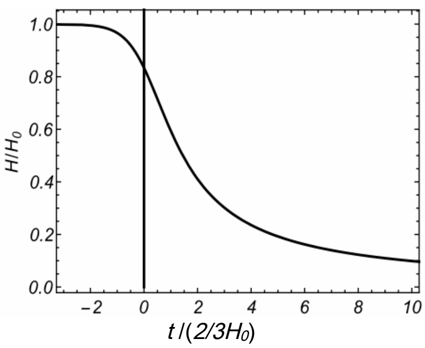

Fig. 2. The rescaled Hubble function, H/H_0 vs t.

Taking into account (13), for the time derivative of the Hubble function one finds

$$\frac{\dot{H}}{H^2} = q - 1 = 3\left[\left(\frac{H}{H_0}\right)^2 - 1\right]. \tag{14}$$

In Fig. 1 is plotted the function $V(\Phi)$. The parameter q becomes zero at the moment $t=0$, which practically corresponds to the inflection point of the potential $V(\Phi)$. At this point the exponential behavior of the scale factor is changed to the power-law one.

This shows that at the intitial stage of the inflation one has $q=1$. For the acceleration parameter near zero we have $\left|\dot{H}/H^2\right| = 3\dot{\Phi}^2/(2V) \approx 1$, which means that one of the conditions for slow-roll is violated. As regards the potential of the scalar field, one gets

$$V(q) = \frac{3y_0}{8\pi}H_0^2\left(\frac{H}{H_0}\right)^4 = \frac{3y_0}{8\pi}H_0^2\left(\frac{q+2}{3}\right)^2. \tag{15}$$

For $q=0$ this gives $8\pi V/y_0 = 4H_0^2/3$, which, again, violates the slow-roll conditions [4–6]. Indeed, the form of the Eq. (3) allows a simple physical interpretation – the mechanical rolling in the potential $V(\Phi)$ in the presence of a time-dependent Hubble friction coefficient $3H$. The inflation in the regime of slow rolling starts, when the term $3H\dot{\Phi}$ dominates the acceleration term, $|\ddot{\Phi}/(3H\dot{\Phi})| \ll 1$. The Hubble friction leads to the shift of Φ in the direction of decreasing potential. The second condition for the slow roll is given by $\dot{\Phi}^2/(2V(\Phi)) \ll 1$. These conditions lead to the effective eqaution of state $P_{eff} \approx -\rho_{eff}$, which corresponds to the vacuum like matter typical for the inflation.

Under the slow roll conditions, the set of cosmological equations takes the form

$$3H\dot{\Phi} \approx -V',\ 3H^2 \approx \frac{8\pi}{y_0}V,\ \ 2\dot{H}+3H^2 \approx \frac{8\pi}{y_0}V. \tag{16}$$

Estimation of the Inflation Duration and the Total Number of *e*-Foldings. The obvious inhomogeneity of the recent Universe and the observed anisotropy of the CMB radiation indicates the generation of density inhomogeneities in the early stages in the early stages of the cosmological expansion, having the order $\delta\rho/\rho \sim 10^{-5}$ and with the spectrum close to the flat Harrison–Zeldovich power spectrum [7]. In the model model under consideration, the expansion is driven by the above mentioned potential αH^4 operates up to the value of the scalar $\Phi = \Phi_c$. After this, the secondary potential of the form $k(\Phi-\Phi_c)^n/2$ dominates. The suggested hybrid inflation ends by small oscillations near the minimum of the secondary potential when the kinetic energy, as it seen from (15), is comparable to the potential energy. The Hubble friction is subdominant, and the rapid oscillations lead to the creation of particles and the transition to the hot radiation dominated stage takes place [8].

The duration of the inflation within the framework of the problem at hand was already determined as

$$\Delta t_{\inf} = \frac{1}{3}H_0^{-1}. \tag{17}$$

Taking into account that the value of the Hubble parameter during the inflationary stage is estimated as [9] $10^{42}s^{-1} > H_0 > 10^{36}s^{-1}$, from (17) we obtain the duration of the inflationary stage. For the number of *e*-foldings, defined as $N(\Phi) = \ln(a_k/a(\Phi))$ ($a(\Phi)$ is the the scale factor as a function of the inflaton field Φ and a_k is the value of the scale factor at the end of the inflation), in the "slow roll" regime, by taking into account (7), one has

$$N(\Phi) = \int_{\Phi}^{\Phi_e} H(\Phi)\frac{d\Phi}{\dot{\Phi}} = \frac{8\pi}{y_0}\int_{\Phi}^{\Phi_e}\frac{H(\Phi)}{2H'}d\Phi = \frac{1}{3}\ln\left(\frac{\sinh\Phi_e}{\sinh\Phi}\right). \tag{18}$$

Under the slow-roll conditions

$$\varepsilon = \frac{y_0}{16\pi}\left(\frac{V'}{V}\right)^2 \ll 1,\ \eta = \frac{y_0}{8\pi}\left|\frac{V''}{V}\right| \ll 1,$$

the necessary value $N(\Phi)$ is formed well before the end of inflation.

Conclusion. It is known that the inflationary paradigm provides a natural solution of the flatness and horizon problems in standard cosmology and generates seeds for density perturbations, which are the source for the large-scale structure formation in the Universe. These perturbations are caused by quantum fluctuations of the field, responsible for the inflation (inflaton). One of the most important predictions of the inflation is that the density perturbations have nearly scale invariant spectrum. It is noteworthy that this prediction can be directly verified by the measurements of the temperature anisotropies of the CMB radiation. The temperature anisotropies registered by various observational projects (COBE, WMAP, Planck) confirm the main predictions of the inflationary scenario within the observational accuracy. Thus, the properties of the early Universe can be studied with high-precision observations. The model with a quartic potential with a non-minimal gravitational interaction in the Jordan frame is in full agreement with the data from PLANK satellite [10].

In this paper we have studied the inflationary dynamics in the presence of a scalar field using the modified Jordan–Brans–Dicke theory. The role of the inflatonary potential $V(\Phi) = \alpha H^4$ is considered in the Einstein frame of the Jordan theory. A model is constructed for a hybrid inflation, the completion of which corresponds to the vanishing of the acceleration parameter q.

This work was supported by State Committee of Science MES RA, in frame of research project SCS "15T-1C110".

Received 06.07.2016

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