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ON THE NUMBER OF VERTICES WITH AN INTERVAL SPECTRUM IN EDGE LABELING OF REGULAR GRAPHS

N. N. DAVTYAN[1]*, R. R. KAMALIAN[1,2]**

[1] *Chair of Programming and Information Technologies of Ijevan Branch of the YSU, Armenia*

[2] *The Institute for Informatics and Automation Problems of NAS of the Republic of Armenia*

Undirected simple finite graphs are considered. An upper bound of the number of vertices with an interval spectrum is obtained for any edge labeling of an arbitrary regular graph.

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Introduction. We consider undirected simple finite graphs. The sets of vertices and edges of a graph G are denoted by $V(G)$ and $E(G)$ respectively. For a graph G we denote by $\delta(G)$ the least degree of a vertex of G. For any graph G we define a parameter $c(G)$ in the following way: if G is empty, then $c(G) \equiv 0$, otherwise, $c(G)$ is equal to the number of connected components of G. If G is a graph, $x \in V(G)$, $y \in V(G)$, then $d_G(x,y)$ denotes the distance between the vertices x and y in G. If G is a graph, $x \in V(G)$ and $V_0 \subseteq V(G)$, then $d_G(x,V_0)$ denotes the distance in the graph G between its vertex x and the subset V_0 of its vertices. For a graph G and an arbitrary subset $V_0 \subseteq V(G)$ $G[V_0]$ denotes the subgraph of the graph G induced by the subset V_0 of its vertices.

For any graph G and its arbitrary subgraph H let us define the subgraph $S[H,G]$ of the graph G as follows:

$$V(S[H,G]) \equiv \{x \in V(G)/\ d_G(x,V(H)) \leq 1\},$$
$$E(S[H,G]) \equiv E(H) \cup \{(x,y) \in E(G)/\ x \in V(S[H,G]) \backslash V(H), y \in V(H)\}.$$

An arbitrary nonempty finite subset of consecutive integers is called an interval. A bijection $\varphi : E(G) \to \{1,2,\ldots,|E(G)|\}$ is called an edge labeling of the graph G. For a graph G the set of all its edge labelings is denoted by $\tau(G)$.

* E-mail: nndavtyan@gmail.com ** E-mail: rrkamalian@yahoo.com

If G is a graph, $x \in V(G)$, $\varphi \in \tau(G)$, then the set

$$S_G(x,\varphi) \equiv \{\varphi(e)/e \in E(G),\ e \text{ is incident with } x\}$$

is called a spectrum of the vertex x of the graph G for its edge labeling φ. If G is a graph, $\varphi \in \tau(G)$, then $V_{int}(G,\varphi) \equiv \{x \in V(G)/\ S_G(x,\varphi) \text{ is an interval}\}$.

The terms and concepts, which are not defined can be found in [1].

An upper bound for the cardinality of the set $V_{int}(G,\varphi)$ is obtained in the case when G is a regular graph with $\delta(G) \geq 2$ and $\varphi \in \tau(G)$.

The Main Results. First we recall the following

Proposition [2]. Let G be a graph with $\delta(G) \geq 2$. Let $\varphi \in \tau(G)$ and $V_{int}(G,\varphi) \neq \emptyset$. Then $G[V_{int}(G,\varphi)]$ is a forest, each connected component of which is a simple path.

Theorem. If G is a r-regular graph, $r \geq 2$, $\varphi \in \tau(G)$, then

$$|V_{int}(G,\varphi)| \leq \left\lfloor \frac{r|V(G)| - 2c(G[V_{int}(G,\varphi)])}{2(r-1)} \right\rfloor.$$

Proof. Let $c(G[V_{int}(G,\varphi)]) = k$.

Case 1. $V_{int}(G,\varphi) = \emptyset$.

In this case the required inequality is the following evident one:

$$0 \leq \left\lfloor \frac{r|V(G)|}{2(r-1)} \right\rfloor.$$

Case 2. $V_{int}(G,\varphi) \neq \emptyset$.

In this case $k \geq 1$. Since $\delta(G) = r \geq 2$, by Proposition, $G[V_{int}(G,\varphi)]$ is a forest with k connected components and each of these components is a simple path.

Let $P_1,\ldots,P_k$ be all the connected components of the forest $G[V_{int}(G,\varphi)]$.

It is not difficult to see that for $\forall i$, $1 \leq i \leq k$, the equality

$$|E(S[P_i,G])| = (r-1)V(P_i)| + 1$$

holds.

Let us also note that (if $k \geq 2$) for arbitrary integers i' and i'' satisfying the inequality $1 \leq i' < i'' \leq k$, the relation $E(S[P_{i'},G]) \cap E(S[P_{i''},G]) = \emptyset$ holds.

Taking into account the evident relation $(\bigcup_{i=1}^{k} E(S[P_i,G])) \subseteq E(G)$, we obtain

$$|E(G)| = \frac{r|V(G)|}{2} \geq \left| \bigcup_{i=1}^{k} E(S[P_i,G]) \right| = \sum_{i=1}^{k} |E(S[P_i,G])| =$$

$$= \sum_{i=1}^{k} ((r-1)|V(P_i)| + 1) = k + (r-1)\sum_{i=1}^{k} |V(P_i)| = k + (r-1)|V_{int}(G,\varphi)|,$$

$$|V_{int}(G,\varphi)| \leq \frac{1}{r-1}\left(\frac{r|V(G)|}{2} - k\right) = \frac{r|V(G)| - 2k}{2(r-1)}.$$

Consequently,

$$|V_{int}(G,\varphi)| \leq \left\lfloor \frac{r|V(G)| - 2k}{2(r-1)} \right\rfloor. \qquad \square$$

C o r o l l a r y 1. If G is a r-regular graph, $r \geq 2$, $\varphi \in \tau(G)$, then

$$|V_{int}(G,\varphi)| \leq \left\lfloor \frac{r|V(G)|-2}{2(r-1)} \right\rfloor.$$

C o r o l l a r y 2. If G is a cubic graph, $\varphi \in \tau(G)$, then

$$|V_{int}(G,\varphi)| \leq \left\lfloor \frac{3|V(G)|-2c(G[V_{int}(G,\varphi)])}{4} \right\rfloor.$$

C o r o l l a r y 3. If G is a cubic graph, $\varphi \in \tau(G)$, then

$$|V_{int}(G,\varphi)| \leq \left\lfloor \frac{3|V(G)|-2}{4} \right\rfloor.$$

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R E F E R E N C E S

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