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PROBLEM OF OPTIMAL STABILIZATION UNDER INTEGRALLY SMALL PERTURBATIONS

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In the present work the optimal stabilization problem of a moving mass center of satellite under influence of integrally small perturbations during finite time intervals has been considered. The optimal stabilization problem of the above motion in classical sense and under integrally small perturbations is assumed and respectively solved. A comparison between the optimal values of performance indices in mentioned cases proves that the energy consumption at stabilization under integrally small perturbations is less than stabilization in classical sense.

Keywords: optimal stabilization, optimal control, dynamical systems, perturbation.

Introduction. In optimal control problems a task is set of finding a control law for a given system such that a certain optimality criterion is achieved. A control problem includes the determination of a performance index that is a function of state and control variables. An optimal control is a set of differential equations describing the paths of control variables that minimizes the performance index.

In addition, the stability analysis and stabilization of nonlinear systems are among important and extensively studied problems in the control theory. The Lyapunov function based method played an important role in providing solutions to these problems.

In the present paper the optimal stabilization problem in the mass centre of satellite in motion in the classical sense and under integrally small perturbations is raised and respectively solved. Finally the results of mentioned solutions were compared.

Differential Equations of Perturbed Motion of the Mass Centre of Satellite. The optimal stabilization problem under integrally small perturbations has been studied for dynamical systems. In the case of obtained sufficiency conditions there is a solution for the problem of optimal stabilization for such systems, and an algorithm has been constructed for not fully controllable linear systems [1].

Let consider of perturbed motion of mass center of satellite assuming that the satellite is affected only by the Earth gravity. The reduced resultant force F,

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applied to the mass center of satellite [2], is determined by the universal law of gravitation

$$F = \mu m / r^2, \tag{1}$$

where $\mu = gR^2 = fM$ is the standard gravitational parameter of the Earth (R is the radius; g is the gravitational acceleration at the Earth surface; M is the Earth mass; f is gravitational constant); $r = OC$ is the distance from the center of Earth O to the mass center of satellite C; m is the satellite mass.

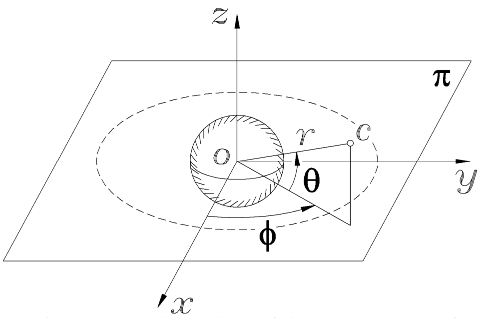

Fig. 1. Steady motion of the mass center of satellite.

We consider the steady motion of the mass center of satellite in a circular orbit of radius r_0 lying in π plane. The motion of this kind is also called the stationary steady motion of satellite (Fig. 1). The defined parameters of the steady motion of satellite must satisfy the following condition that directly follows from the Newton's second law:

$$\omega^2 r_0^3 = \mu \quad (m r_0 \omega^2 = \mu m / r_0^2), \tag{2}$$

where $\omega = \dot{\varphi} = \text{const}$ is the angular velocity of satellite rotation with radius vector r_0 at the steady motion.

Now consider this satellite motion under influence of some perturbations (that is equivalent to satellite separation from the rocket at the last stage along with slight change of conditions that should ensure the motion of satellite in a circular orbit of radius r_0 in π plane). As a result of imposed perturbations the motion of satellite shall be perturbed, in particular, changing the orbit to a non-circular in a plane other than in π plane and rotational angular velocity $\dot{\varphi}$ of the radius vector being not equal to $\sqrt{\mu / r_0^3}$.

To set up the equations of perturbed satellite motion, a reference system $Oxyz$ is constructed, the coordinate plane xy, of which is adjusted to the orbital plane of steady motion, i.e. to the π plane. The centre of gravity of the satellite C in the perturbed motion will be defined by the spherical coordinates r, φ, θ (Fig. 1).

By setting $r = r_0 + x$ and $\dot{\varphi} = \omega + y$, we obtain the equations of perturbed motion of satellite (the equations for θ and φ are similar) [2].

For generality, a new notation $x = x_1$, $\dot{x} = x_2$, $\theta = x_3$, $\dot{\theta} = x_4$, $y = x_5$ is introduced and the differential equations of perturbed satellite motion in the first approximation are:

$$\dot{x}_1 = x_2, \ \dot{x}_2 = 3\omega^2 x_1 + 2 r_0 \omega x_5, \ \dot{x}_3 = x_4, \ \dot{x}_4 = -2\omega^2 x_3, \ \dot{x}_5 = (-2\omega / r_0) x_2. \tag{3}$$

The above equations were derived with due regard for the equality in (2).

Formulation and Solution of Optimal Stabilization Problem in the Classical Sense. For investigation of stability by means of Lyapunov indirect method, the differential equations of above motion are taken to form a system of differential equations (3), and the characteristic equation for this system of differential equations is as follows:

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 3\omega^2 & 0 & 0 & 0 & 2r_0\omega \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -2\omega^2 & 0 & 0 \\ 0 & -2\omega/r_0 & 0 & 0 & 0 \end{bmatrix}. \tag{4}$$

The eigenvalues of A_1 will be

$$\lambda_1 = 0, \quad \lambda_{2,3} = \pm i\omega, \quad \lambda_{4,5} = \pm i\sqrt{2}\omega. \tag{5}$$

So, the system is marginally stable in the Lyapunov sense, but unstable under integrally small perturbations [3].

Let consider the input controls $\overline{u}_1$ and $\overline{u}_2$ in the φ and θ generalized coordinate directions respectively, then first approximation of differential equations of the perturbed motion of satellite are obtained

$$\begin{cases} \dot{x}_1 = x_2, \quad \dot{x}_2 = 3\omega^2 x_1 + 2r_0\omega x_5, \quad \dot{x}_3 = x_4, \\ \dot{x}_4 = -2\omega^2 x_3 + (\overline{u}_1 / r_0), \quad \dot{x}_5 = -(2\omega / r_0) x_2 + (\overline{u}_2 / r_0^2). \end{cases} \tag{6}$$

At the derivation of above equations the equality in (2) was taken into account.

Now make the following notations:

$$y_1 = (1/r_0)x_1, \; y_2 = (\sqrt{1/r_0 g})x_2, \; y_3 = x_3, \; y_4 = (\sqrt{r_0/g})x_4, \; y_5 = (\sqrt{r_0/g})x_5. \tag{7}$$

Then, the system is written in a dimensionless form

$$\dot{y}_1 = ay_2, \; \dot{y}_2 = 3by_1 + 2y_5, \; \dot{y}_3 = ay_4, \; \dot{y}_4 = -2by_3 + u_1, \; \dot{y}_5 = -2y_2 + u_2. \tag{8}$$

Here

$$\begin{gathered} \dot{y}_i = dy_i / dt', \; t' = \omega t, \, u_1 = \overline{u}_1 / (\omega\sqrt{r_0 g}), \; u_2 = \overline{u}_2 / (\omega\sqrt{r_0 g}), \\ a = (1/\omega)\sqrt{g / r_0}, \quad b = 1/a. \end{gathered} \tag{9}$$

The system of differential equations (8) is fully controllable as

$$\operatorname{rank} K = \operatorname{rank}\left[B_2, \, A_2B_2, \, A_2^2B_2, \; A_2^3B_2, \, A_2^4B_2 \right] = 5, \tag{10}$$

where

$$A_2 = \begin{bmatrix} 0 & a & 0 & 0 & 0 \\ 3b & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & a & 0 \\ 0 & 0 & -2b & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{11}$$

The problem of optimal stabilization of the system is solved when minimizing the performance index

$$J[u] = \int_0^\infty \left(\sum_{i=1}^{5} y_i^2 + \sum_{k=1}^{2} u_k^2 \right) dt. \tag{12}$$

Let set up an expression

$$B[u] = \frac{\partial V}{\partial t} + \sum_{i=1}^{5} \frac{\partial V}{\partial y_i} \dot{y}_i + \sum_{i=1}^{5} y_i^2 + \sum_{k=1}^{2} u_k^2, \tag{13}$$

then, since the expression in (13) at optimal control takes the minimum value equal to zero [4, 5],

$$\begin{gathered} B\big|_{u=u^0} = 0, \qquad B[V, y_1, y_2, y_3, y_4, y_5, u_1^0, u_2^0] = \\ = \frac{\partial V}{\partial y_1}(ay_2) + \frac{\partial V}{\partial y_2}(3by_1 + 2y_5) + \frac{\partial V}{\partial y_3}(ay_4) + \frac{\partial V}{\partial y_4}(-2by_3 + u_1) + \\ + \frac{\partial V}{\partial y_5}(-2y_2 + u_2) + y_1^2 + y_2^2 + y_3^2 + y_4^2 + y_5^2 + u_1^{02} + u_2^{02} = 0 \end{gathered} \tag{14}$$

and

$$\left.\frac{\partial B}{\partial u}\right|_{u_i = u_i^0} = 0 \quad (i = 1, 2), \tag{15}$$

where $u = [u_1 \quad u_2]^T$ and u_i^0 are optimal controls.

For Lyapunov function we will search the solution in the following form:

$$V = \frac{1}{2} \sum_{i,j=1,2,5} c_{ij} y_i y_j + \frac{1}{2} \sum_{i,j=3,4} c_{ij} y_i y_j , \tag{16}$$

where c_{ij} are constants.

From equation (15) we obtain

$$u_1^0 = -\partial V / 2\partial y_4; \quad u_2^0 = -\partial V / 2\partial y_5. \tag{17}$$

Substituting the values u_i^0 from equation (17) in (13), considering the equations (14) and (16), a set of equations for determination of c_{ij} constants is obtained

$$\begin{cases} 3bc_{12} - \frac{c_{15}^2}{4} + 1 = 0, & \frac{ac_{11}}{2} + \frac{3bc_{22}}{2} - c_{15} - \frac{c_{15}c_{25}}{4} = 0, \quad c_{12} + \frac{3bc_{25}}{2} - \frac{c_{15}c_{55}}{4} = 0, \\ ac_{12} - \frac{c_{25}^2}{4} - 2c_{25} + 1 = 0, & \frac{ac_{15}}{2} + c_{22} - c_{55} - \frac{c_{25}c_{55}}{4} = 0, \quad -\frac{c_{34}^2}{4} - 2bc_{34} + 1 = 0, \\ \frac{ac_{33}}{2} - bc_{44} - \frac{c_{34}c_{44}}{4} = 0, & ac_{34} - \frac{c_{44}^2}{4} + 1 = 0, \quad 2c_{25} - \frac{c_{55}^2}{4} + 1 = 0. \end{cases} \tag{18}$$

Solving these equations at $\omega = 0.001\, s^{-1}$; $g = 9.81\, m \cdot s^{-2}$; $r_0 = 7000000\, m$ we get the solution that satisfies the conditions of the optimal stabilization problem:

$$\begin{gathered} c_{11} = 27.096; \quad c_{12} = 13.019; \quad c_{15} = 11.661; \quad c_{22} = 8.131; \quad c_{25} = 5.036; \\ c_{33} = 4.258; \quad c_{34} = 0.548; \quad c_{44} = 2.568; \quad c_{55} = 6.655. \end{gathered} \tag{19}$$

Thus, optimal Lyapunov function will have the form

$$\begin{gathered} V^0(y_1, ..., y_5) = 13.548 y_1^2 + 4.068 y_2^2 + 2.129 y_3^2 + 1.284 y_4^2 + 3.328 y_5^2 + \\ + 13.019 y_1 y_2 + 11.661 y_1 y_5 + 5.036 y_2 y_5 + 0.548 y_3 y_4 \end{gathered} \tag{20}$$

and optimal controls will be

$$u_1^0 = -0.274y_3 - 1.284y_4, \quad u_2^0 = -5.830y_1 - 2.518y_2 - 3.328y_5. \qquad (21)$$

For the optimal value of performance index in equation (12) we obtain

$$\begin{gathered} J^0 = V^0(y_{10},\dots,y_{50}) = 13.548y_{10}^2 + 4.068y_{20}^2 + 2.129y_{30}^2 + 1.284y_{40}^2 + \\ +3.328y_{50}^2 + 13.019y_{10}y_{20} + 11.661y_{10}y_{50} + 5.036y_{20}y_{50} + 0.548y_{30}y_{40}, \end{gathered} \qquad (22)$$

where $y_{i0} = y_i(0)$, $i = 1,\dots,5$.

Formulation and Solution of Optimal Stabilization Problem Under Integrally Small Perturbations. Let consider the input controls $\bar{u}_1$ and $\bar{u}_2$ in the φ and r generalized coordinate directions respectively, then first approximation of differential equations of the perturbed motion of satellite are obtained:

$$\begin{cases} \dot{x}_1 = x_2, \quad \dot{x}_2 = 3\omega^2 x_1 + 2r_0\omega x_5 + \bar{u}_2, \quad \dot{x}_3 = x_4, \\ \dot{x}_4 = -2\omega^2 x_3 + (\bar{u}_1 / r_0), \quad \dot{x}_5 = (-2\omega / r_0) x_2. \end{cases} \qquad (23)$$

Let use notations of equations (7) and write the mentioned system with dimensionless sizes

$$\dot{y}_1 = ay_2, \quad \dot{y}_2 = 3by_1 + 2y_5 + u_2, \quad \dot{y}_3 = ay_4, \quad \dot{y}_4 = -2by_3 + u_1, \quad \dot{y}_5 = -2y_2. \qquad (24)$$

Here

$$\begin{gathered} \dot{y}_i = \frac{dy_i}{dt'}, \quad t' = \omega t, \quad u_1 = \bar{u}_1 \big/ (\omega\sqrt{r_0 g}), \quad u_2 = \bar{u}_2 \big/ (\omega\sqrt{r_0 g}), \\ a = (1/\omega)\sqrt{g / r_0}, \quad b = 1/a. \end{gathered} \qquad (25)$$

Let replace the following problem.

Finds such input controls u_1^0 and u_2^0, which will ensure the stability of the solution y_i=0 (i=1,…,5) of the system of differential equations (24) under integrally small perturbations and will minimize the performance index.

The system (24) is not fully controllable as

$$\operatorname{rank} K_1 = \operatorname{rank}[B_3, A_3B_3, A_3^2B_3, A_3^3B_3, A_3^4B_3] = 4, \qquad (26)$$

where

$$A_3 = \begin{bmatrix} 0 & a & 0 & 0 & 0 \\ 3b & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & a & 0 \\ 0 & 0 & -2b & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}. \qquad (27)$$

As the system (24) is not fully controllable, hence, the optimal stabilization problem for the system is not solved in the sense of [5].

Since the linear transformation does not change the eigenvalues of the matrix, the characteristic equation corresponding to system of differential equations (24) has one zero root. It means that the system (24) admits the first integral. It is known [6], that in this case it is possible with help of nonspecial linear transformation $Z = CY$ ($\det C \neq 0$, $Z = [z_1 \dots z_2]^T$, $Y = [y_1 \dots y_2]^T$),

$$z_1 = -3by_1 + y_2 - 2y_5,\; z_2 = 3by_1 + y_2 + 2y_5,\; z_3 = y_3,\; z_4 = y_4,\; z_5 = 2by_1 + y_5 \quad (28)$$

to reduce the system (24) to the form of

$$\dot{z}_1 = z_2 + u_2,\; \dot{z}_2 = -z_1 + u_2,\; \dot{z}_3 = az_4,\; \dot{z}_4 = -2bz_3 + u_1,\; \dot{z}_5 = 0. \quad (29)$$

For system of differential equations (29) the optimal stabilization problem may be solved under integrally small perturbations [1]. The minimized performance index shall be adopted in the form of

$$J_1[u] = \int_0^\infty \left(\sum_{i=1}^4 z_i^2 + \sum_{i=1}^2 u_i^2 \right) dt. \quad (30)$$

Thus, for system (29) it is required to resolve the optimal stabilization problem under integrally small perturbations, while minimizing the performance index in (30).

Now let write the expression of Bellman for the system (29):

$$\mathbf{B}[u] = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial z_1}(z_2 + u_2) + \frac{\partial V}{\partial z_2}(-z_1 + u_2) + \frac{\partial V}{\partial z_3}(az_4) + \frac{\partial V}{\partial z_4}(-2bz_3 + u_1) + \\ + z_1^2 + z_2^2 + z_3^2 + z_4^2 + u_1^2 + u_2^2. \quad (31)$$

Since at optimal control the expression in (31) takes the minimum value equal to zero [1], then we obtain

$$u_1^0 = -\partial V / 2\partial z_4;\; u_2^0 = -\partial V / 2\partial z_1 - \partial V / 2\partial z_2. \quad (32)$$

By substituting equation (32) into (31), we obtain

$$\mathbf{B}[u] = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial z_1}(z_2) + \frac{\partial V}{\partial z_2}(-z_1) + \frac{\partial V}{\partial z_3}(az_4) + \frac{\partial V}{\partial z_4}(-2bz_3) + z_1^2 + z_2^2 + z_3^2 + z_4^2 - \\ - \frac{1}{4}\left(\frac{\partial V}{\partial z_4}\right)^2 - \frac{1}{4}\left(\frac{\partial V}{\partial z_1} + \frac{\partial V}{\partial z_2}\right)^2. \quad (33)$$

For Lyapunov function we will search the solution in the following form [1]:

$$V(t,z) = V_2(z) + V_1(t,z) + V_0(t), \quad (34)$$

where $V_2(z)$ is the quadratic form with constant coefficients; $V_1(t, z)$ is the first degree form with respect to z with time dependent coefficients; $V_0(t)$ is the function of time. Substituting equation (34) into (33), we can write after some simple transformations

$$\frac{\partial V_1(t,z)}{\partial t} + \frac{\partial V_0(t)}{\partial t} + \frac{\partial V_2(z)}{\partial z_1}(z_2) + \frac{\partial V_1(t,z)}{\partial z_1}(z_2) + \frac{\partial V_2(z)}{\partial z_2}(-z_1) + \frac{\partial V_1(t,z)}{\partial z_2}(-z_1) + \\ + \frac{\partial V_2(z)}{\partial z_3}(az_4) + \frac{\partial V_1(t,z)}{\partial z_3}(az_4) + \frac{\partial V_2(z)}{\partial z_4}(-2bz_3) + \frac{\partial V_1(t,z)}{\partial z_4}(-2bz_3) + \\ + z_1^2 + z_2^2 + z_3^2 + z_4^2 - \frac{1}{4}\left(\frac{\partial V_2(z)}{\partial z_4} + \frac{\partial V_1(t,z)}{\partial z_4} + \frac{\partial V_0(t)}{\partial z_4}\right)^2 - \\ - \frac{1}{4}\left(\frac{\partial V_2(z)}{\partial z_1} + \frac{\partial V_1(t,z)}{\partial z_1} + \frac{\partial V_0(t)}{\partial z_1} + \frac{\partial V_2(z)}{\partial z_2} + \frac{\partial V_1(t,z)}{\partial z_2} + \frac{\partial V_0(t)}{\partial z_2}\right)^2 = 0. \quad (35)$$

Based on definitions of functions V_2, V_1, V_0, we have $V_1 = V_0 = 0$, and equation (33) will be

$$\frac{\partial V_2(z)}{\partial z_1}(z_2) + \frac{\partial V_2(z)}{\partial z_2}(-z_1) + \frac{\partial V_2(z)}{\partial z_3}(az_4) + \frac{\partial V_2(z)}{\partial z_4}(-2bz_3) + \\ + z_1^2 + z_2^2 + z_3^2 + z_4^2 - \frac{1}{4}\left(\frac{\partial V_2(z)}{\partial z_4}\right)^2 - \frac{1}{4}\left(\frac{\partial V_2(z)}{\partial z_1} + \frac{\partial V_2(z)}{\partial z_2}\right)^2 = 0. \tag{36}$$

For function $V_2(z)$ we can search the solution in the form

$$V_2(z) = \frac{1}{2}\sum_{i,j=1}^{2} c_{ij} z_i z_j + \frac{1}{2}\sum_{i,j=3}^{4} c_{ij} z_i z_j .$$

Then the system of equations for c_{ij} constants determination will be

$$\begin{cases} -\dfrac{(c_{11}+c_{12})^2}{4} - c_{12} + 1 = 0, & \dfrac{c_{11}}{2} - \dfrac{(c_{11}+c_{12})(c_{12}+c_{22})}{4} - \dfrac{c_{22}}{2} = 0, \\ c_{12} - \dfrac{(c_{12}+c_{22})^2}{4} + 1 = 0, & -\dfrac{c_{34}^2}{4} - 2bc_{34} + 1 = 0, \\ \dfrac{ac_{33}}{2} - \dfrac{c_{34}c_{44}}{4} - bc_{44} = 0, & ac_{34}\dfrac{c_{44}^2}{4} + 1 = 0. \end{cases} \tag{37}$$

So, the solution of the mentioned system at r_0=7000000 m; ω=0.001 s^{-1} will be

$$c_{11} = 3.275;\ c_{12} = -0.681;\ c_{22} = 1.810;\ c_{33} = 4.258;\ c_{34} = 0.548;\ c_{44} = 2.568. \tag{38}$$

Thus, optimal Lyapunov function will be

$$V^0(z_1, z_2, z_3, z_4) = 1.637z_1^2 + 0.905z_2^2 + 2.129z_3^2 + \\ + 1.284z_4^2 - 0.681z_1z_2 + 0.548z_3z_4, \tag{39}$$

and optimal controls will be

$$u_1^0 = -0.274z_3 - 1.284z_4, \quad u_2^0 = -1.2974z_1 - 0.565z_2. \tag{40}$$

We obtain for the optimal value of performance index in equation (30):

$$J_1^0 = V^0(z_{10}, z_{20}, z_{30}, z_{40}) = 1.637z_{10}^2 + 0.905z_{20}^2 + 2.129z_{30}^2 + 1.284z_{40}^2 - \\ - 0.681z_{10}z_{20} + 0.548z_{30}z_{40}, \tag{41}$$

where $z_{i0} = z_i(0),\ i = 1,...,4$.

As the small vibrations of the integral acting on the system are unknown, it means that within the given interval they have their influence, and it is impossible to select u control in its optimal way. That is why the optimal controls u_1^0 and u_2^0 obtained in the result of the problem solution are constructed beginning from the moment $t_0 = 0$.

Results and Discussion. The value of performance index (12) at the usual stabilization in equation (22) is obtained.

In addition, taking into account the transformation $Z = CY$ in (28) and the optimal value of performance index in (41), the value of performance index at stabilization under integrally small perturbations is as follows:

$$J_1^0 = V^0(y_{10},...,y_{50}) = 20.703y_{10}^2 + 1.861y_{20}^2 + 2.129y_{30}^2 + 1.284y_{40}^2 + \\ + 12.892y_{50}^2 - 3.710y_{10}y_{20} + 32.678y_{10}y_{50} - 2.928y_{20}y_{50} + 0.548y_{30}y_{40}, \quad (42)$$

where $y_{i0} = y_i(0),\ \ i = 1,...,5$.

Conclusion. Based on comparison of the values of performance indices in equations (22) and (42) it is proved that $J_1^0 < J^0$.

It was shown that the energy consumption at the stabilization in the sense given in [4, 5] is more than at stabilization under integrally small perturbations.

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