

DIAMAGNETISM IN THE CYLINDRICAL QUANTUM DOT WITH PARABOLIC CONFINEMENT POTENTIAL

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Diamagnetic properties of the electron gas in a cylindrical quantum dot with parabolic confinement potential have been investigated. The analytical expressions have been obtained for mean energy, mean magnetization and mean magnetic susceptibility of the electron gas. The diamagnetic character of such system has been shown.

Keywords: cylindrical quantum dot, parabolic confinement potential, quantum dot diamagnetism.

1. Introduction. The electron gas localized in the quantum dot may exhibit interesting properties purely inherent to low-dimensional structures. A striking example of what has been said is the generalization of Kohn's theorem [1] for the cases of quantum wells and dots [2–5]. It is clear that the multi-electron systems can demonstrate the statistical properties that can be controlled by changing the geometric dimensions of the studied nanostructures [6–8]. Particularly, in the paper [6] have been studied the thermodynamic and magnetic properties of the cylindrical QD with asymmetrical confinement potential. It has been shown that the magnetic properties of the system reveal the paramagnetic behavior of the electron gas in the QD.

In this paper has been studied the diamagnetism of electron gas localized in the cylindrical QD with parabolic confinement potential. The undisputed advantage of a chosen model of QD is the exact solvability of it. This makes it possible to obtain a lot of analytic results.

2. Theory. Let us consider the one-electron states in cylindrical QD with parabolic confinement potential in the external magnetic field.

The Schrodinger equation of such system has the following form

$$\frac{1}{2\mu} \left(\hat{\mathbf{P}} - \frac{e}{c} \mathbf{A} \right)^2 \psi + \hat{V}_{conf}(\rho, \phi, z) \psi = E \psi, \quad (1)$$

where \mathbf{A} is the gauge of magnetic field; μ is the effective mass of electron; $\hat{V}_{conf}(\rho, \phi, z)$ is the confinement potential of QD, which has the following form

$$\hat{V}_{conf}(\rho, \phi, z) = \frac{\mu \omega_\rho^2}{2} \rho^2 + \frac{\mu \omega_z^2}{2} z^2, \quad (2)$$

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where $\omega_\rho \sim \frac{\hbar}{\mu R_0^2}$ and $\omega_z \sim \frac{\hbar}{\mu L^2}$ are the radial and axial frequencies of confinement potential accordingly; R_0 and L are the radius and the height of QD.

We suppose that the magnetic field is directed along the axis of the cylinder. If we choose magnetic gauge as following

$$\mathbf{A} = \left\{ A_\rho = A_z = 0, A_\phi = \frac{B\rho}{2} \right\}, \quad (3)$$

where B is the magnetic field strength, then we can represent Schrodinger equation as

$$-\frac{\hbar^2}{2\mu} \left(\frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial}{\partial \rho} \right) + \frac{\partial^2}{\partial z^2} + \frac{1}{\rho^2} \cdot \frac{\partial^2}{\partial \phi^2} \right) \psi - \frac{i\hbar\omega_B}{2} \cdot \frac{\partial \psi}{\partial \phi} + \frac{\mu\omega_B^2\rho^2}{8} \psi + \hat{V}_{conf}(\rho, \phi, z) \psi = E\psi, \quad (4)$$

where $\omega_B = \frac{eB}{\mu c}$ is the cyclotron frequency.

The solution of (4) has been discussed in [9], and for final wave-function and energy we have the following expressions:

$$\psi(\rho, \phi, z) = \frac{1}{\sqrt{2\pi}} \cdot \frac{1}{a_M^{1+|m|}} \left[\frac{(|m|+n_\rho)!}{2^{|m|}|m|!n_\rho!} \right]^{\frac{1}{2}} e^{im\phi} e^{-\frac{\rho^2}{2a_M^2}} \rho^{|m|} \times \quad (5)$$

$$F_1 \left(-n_\rho, |m|+1, \frac{\rho^2}{2a_M^2} \right) \sqrt{\frac{1}{n_z!2^{n_z}\sqrt{\pi a_z}}} e^{-\frac{z}{2a_z}} H_{n_z} \left(\frac{z}{a_z} \right),$$

$$E_{n_\rho, m, n_z} = \hbar\Omega \left(n_\rho + \frac{|m|+1}{2} \right) + \hbar\omega_B \frac{m}{2} + \hbar\omega_z \left(n_z + \frac{1}{2} \right), \quad (6)$$

where H_{n_z} are Hermite polynomials, $\Omega = \sqrt{\omega_B^2 + (2\omega_\rho)^2}$, $a_M = \sqrt{\frac{\hbar}{\mu\Omega}}$ is magnetic length, $a_z = \sqrt{\frac{\hbar}{\mu\omega_z}}$ is the oscillator length, $n_\rho = 0, 1, 2, \dots$, $m = 0, \pm 1, \pm 2, \dots$ and $n_z = 0, 1, 2, \dots$ are radial, magnetic and axial quantum numbers accordingly.

Assuming that the electron gas described with the Boltzmann statistics, for the partition function [10] we can write

$$Z = \sum_{n_z, n_\rho, m} \exp(-\beta E_{n_\rho, m, n_z}), \quad (7)$$

where $\beta = \frac{1}{k_B T}$ is the inverse temperature.

The partition function of the of investigated system over the discrete energy levels is expressed in the form of triple sum

$$Z = \sum_{n_\rho=0}^{\infty} \sum_{m=-\infty}^{\infty} \sum_{n_z=0}^{\infty} \exp \left(-\beta \left[\hbar\Omega \left(n_\rho + \frac{|m|+1}{2} \right) + \hbar\omega_B \frac{m}{2} + \hbar\omega_z \left(n_z + \frac{1}{2} \right) \right] \right) = \quad (8)$$

$$= \frac{1}{8} \operatorname{csch} \frac{\beta\hbar\omega_z}{2} f^-(\Omega) f^+(\Omega),$$

where $f^\pm(\Omega) = \operatorname{sech} \beta\hbar(\Omega \pm \omega_B)/4$.

With the use of (8) we can calculate the thermodynamic and the magnetic parameters of the considered system.

For mean energy we have

$$\langle E \rangle = -\frac{1}{Z} \cdot \frac{\partial Z}{\partial \beta} = \frac{\hbar \Omega}{4} (g^+(\Omega) + g^-(\Omega)) + \frac{\hbar \omega_B}{4} (g^+(\Omega) - g^-(\Omega)) + \frac{\hbar \omega_z}{2} \coth \frac{\beta \hbar \omega_z}{2}, \quad (9)$$

where $g^\pm(\Omega) = \tanh \beta \hbar (\Omega \pm \omega_B) / 4$.

For mean magnetization

$$\langle M \rangle = \frac{1}{\beta Z} \cdot \frac{\partial Z}{\partial B} = -\frac{\mu_B}{4} \left[\frac{\omega_B}{\Omega} (g^+(\Omega) + g^-(\Omega)) + (g^+(\Omega) - g^-(\Omega)) \right], \quad (10)$$

and for mean magnetic susceptibility we obtain

$$\langle \chi \rangle = \frac{\partial \langle M \rangle}{\partial B} = \left(\frac{\mu_B}{4\Omega} \right)^2 \left[\frac{4}{\hbar} \left(\frac{\omega_B^2}{\Omega} - \Omega \right) (g^+(\Omega) + g^-(\Omega)) - \beta \left((\Omega - \omega_B)^2 f^-(\Omega)^2 + (\Omega + \omega_B)^2 f^+(\Omega)^2 \right) \right], \quad (11)$$

where $\mu_B = \frac{\hbar e}{\mu c}$.

3. Results and Discussion. Fig. 1 shows the electron gas mean energy dependencies on magnetic field strength, when the radius of quantum dot $R_0 = 2a_B^*$. It is seen that the energy is increasing by the increase of magnetic field value. Wherein, the energy growing faster at low temperatures ($T = 100 \text{ K}$, $T = 200 \text{ K}$), and for higher temperatures ($T = 300 \text{ K}$) the growth slows down. From the figure it is clear that the dependencies are not linear, which is related to the presence of QD confinement potential. Note, that at the higher temperatures ($T = 300 \text{ K}$) the energy curve is placed upper than at the lower values of temperature ($T = 100 \text{ K}$, $T = 200 \text{ K}$).

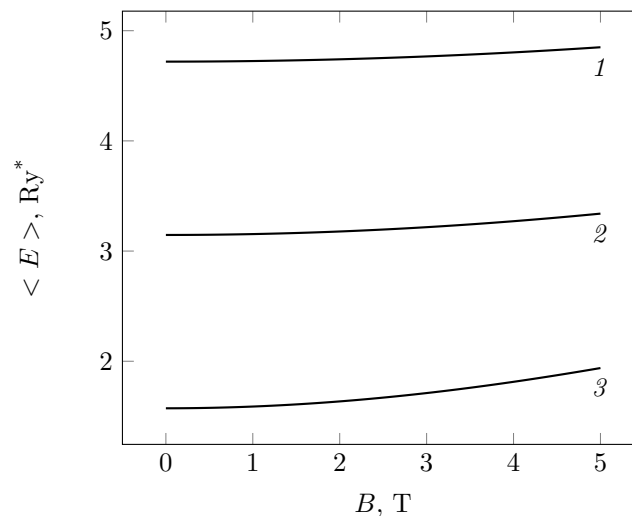


Fig. 1. Dependence of the mean energy on the value of magnetic field ($R_0 = 2a_B^*$):
1. $T = 300 \text{ K}$; 2. $T = 200 \text{ K}$; 3. $T = 100 \text{ K}$.

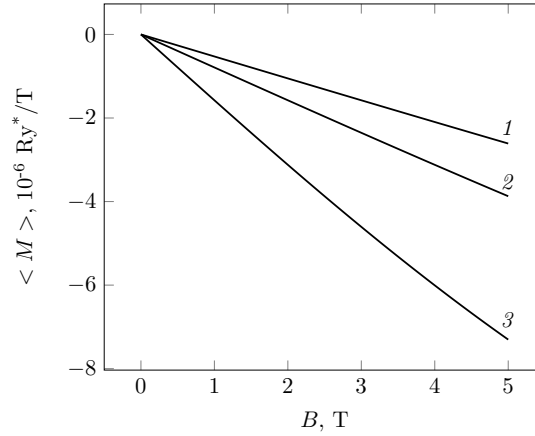


Fig. 2. Dependence of the mean magnetization on the value of magnetic field ($R_0 = 2a_B^*$):
1. $T = 300 K$; 2. $T = 200 K$; 3. $T = 100 K$.

In Fig. 2 it has been shown the electron gas mean magnetization dependencies on magnetic field strength. It follows that the system has the pronounced diamagnetic properties and this dependencies $\langle M(B) \rangle$ are close to the linear. With growth of the temperature, the average magnetization decreases by absolute value because the amplitude of the fluctuations of magnetic moments are increasing, namely, the magnetic moments begin chaotic turns independent from each other, and the total order is becoming weaker. Finally, in the Fig. 3 the dependencies of the electron gas mean magnetic susceptibility on magnetic field strength B are presented. Since $\langle \chi \rangle$ is defined as derivative of $\langle M \rangle$ with respect to B , then taking into the account the almost linear dependency $\langle M(B) \rangle$, the $\langle \chi \rangle$ dependence on the field value is weak. There is a rise of the magnetic susceptibility on the magnetic field at lower temperatures ($T = 100 K$ and $T = 200 K$), which indicates the deviation of $\langle M(B) \rangle$ from the linear law at such temperatures. With increasing the temperature, the dependency $\langle \chi(B) \rangle$ becoming weaker.

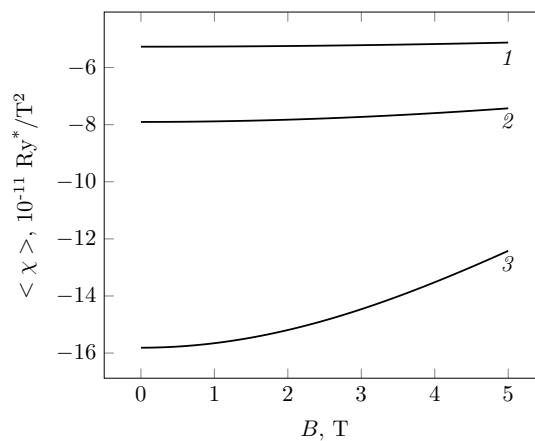


Fig. 3. Dependence of the mean magnetic susceptibility on the value of magnetic field ($R_0 = 2a_B^*$):
1. $T = 300 K$; 2. $T = 200 K$; 3. $T = 100 K$.

4. Conclusion. Thus, the dependencies of mean energy, magnetization and magnetic susceptibility on the strength of the magnetic field at different temperatures for the electron gas localized in the cylindrical QD with parabolic confinement potential has been obtained. The dependence of the mean magnetization on the value of magnetic field shows the pronounced diamagnetic properties of the system. At the relatively low temperatures, there is a slight increase in the magnetic susceptibility on the values of magnetic field, and it practically does not change at higher temperatures.

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