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LETTER TO EDITORIAL GROUP

Mathematics

ON THE SOLUTION OF THE EQUATION $\frac{5}{k}=\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$ ON THE SET OF NATURAL NUMBERS $N\setminus\{60n+1,\, n\in N\}$

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In the present paper it is shown that for every number $k\not\equiv 1 \pmod{60}$, the equation $\frac{5}{k}=\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$ has at least one solution $(x,y,z)\in N$.

MSC2010: 11A67; 11D72.

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V. Sierpinsky in his paper [1] suggested a hypothesis, stating that for any number $k>1$, the equation $5/k=1/x+1/y+1/z$ has a solution (x,y,z) consisting of natural numbers, and the statement was proved in the same paper for all natural numbers $1<k\leq 1000$.

G.Palma has shown that Sierpinsky's hypothesis is true for all natural numbers $k>1$, that are not of the form $1260m+1, m\in N$, as well as for all numbers $k<922322$ [2].

Later other authors have shown that the set of all numbers $k>1$, for which Serpinsky's hypothesis may not be true, is bounded from above, which means that the hypothesis is true for almost all numbers [3–6] .

However, the final solution is still unknown.

The main result of the present paper is the following:

Theorem. If k is a natural number with $k\not\equiv 1 \pmod{60}$, then the equation

$$\frac{5}{k}=\frac{1}{x}+\frac{1}{y}+\frac{1}{z} \tag{1}$$

has a natural solution (x,y,z).

Proof. We will consider are separately several events covering all possible cases of $k\not\equiv 1 \pmod{60}$.

Case 1. If k is an even number, i.e. $k=2n$, then the triple $x=2n$; $y=n$; $z=n$, is a solution for (1).

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Case 2. Assume now that $k \equiv 1 \pmod{10}$. Obviously, $\{10n+1, n \in N\} = \{30n-9, n \in N\} \cup \{30n+1, n \in N\} \cup \{30n-19, n \in N\}$, and we have in this case three possibilities:

Case 2a. If $k = 30n-19$ for some $n \in N$, then the triple $x = 3(2n-1)$; $y = 3(2n-1)(30n-19)$; $z = (2n-1)(30n-19)$ provides a solution of (1).

Case 2b. If $k = 30n-9$ for some $n \in N$, then the triple $x = 6n-1$; $y = 3(6n-1)(30n-3)$; $z = (6n-1)(10n-3)$ is a solution of (1).

Case 2c. Let $k \in \{30n+1, n \in N\}$, it is easy to see that $\{30n+1, n \in N\} = \{60n-29, n \in N\} \cup \{60n+1, n \in N\}$.

Since we have assumed that $k \not\equiv 1 \pmod{60}$, than $k = 60n-29$ for some $n \in N$ and the triple $x = 60n-29$; $y = 15n-7$; $z = (60n-29)(15n-7)$ is the solution for (1).

Case 3. If $k \equiv 3 \pmod{10}$, Then $k = 10n-7$, for some natural n and $x = 2n-1$; $y = (2n-1)(10n-7)$; $z = (2n-1)(10n-7)$ is a solution for (1).

Case 4. Assume $k \equiv 5 \pmod{10}$, so $k = 5(2n-1)$ for some $n \in N$. Then the solution of (1) is $x = 2(2n-1)$; $y = 3(2n-1)$; $z = 6(2n-1)$.

Case 5. Let now $k \equiv 7 \pmod{10}$. We get $k = 10n-3$, then the triple $x = 2n$; $y = 2n(10n-3)$; $z = n(10n-3)$ is solution for (1).

Case 6. Finally, let $k \equiv 9 \pmod{10}$. If $k = 10n-1$, then the following triple provides a solution for (1): $x = 4n$; $y = 4n$; $z = 2n(10n-1)$.

So we show that the equation $5/k = 1/x + 1/y + 1/z$ has natural solutions when $k \notin 60n+1$, $n \in N$ set. $\square$

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