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Physics

MODELING OF THE MAGNETIC FIELD NEAR FLAT-COIL WITH A VIEW TO IMPROVING THE DETECTION EFFICIENCY OF FINE PECULIARITIES OF SUPERCONDUCTIVITY

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Modelling of a RF magnetic field's configuration near the flat coil face of the tunnel diode based single-layer flat-coil-oscillator (SFCO) has been done – that serves as a sensor in one of most of effective methods for study of superconductivity (the SFCO technique). Particularly, the distribution of the measuring magnetic field of flat coil is investigated. An analytical formula has been derived, which allows to determine the parallel and perpendicular to coil face components of the measuring RF magnetic field at any point near the coil. The results of calculations are aimed at an achievement of the optimum sensitivity of the SFCO technique during the experimental study of fine peculiarities of superconductivity that are important for understanding the phenomenon.

Keywords: SFCO technique (a single-layer flat-coil-oscillator method), optimization of experiments, Fulde–Ferrell–Larkin–Ovchinnikov Superconductivity.

Introduction. In the research instrumentation based on the single-layer flat-coil-oscillator (SFCO) devices [1] the frequency and amplitude of low power tunnel diode (TD) harmonic oscillator serve as measuring parameters [2]. The physical quantities during the investigation of superconductivity by means of this technique [3] are determined both by the distortion of the distribution of magnetic field lines of the measuring MHz frequency field of flat coil, and by the energy absorption of the same field [5], as a result of which the frequency or/and amplitude of the oscillator are changed. So, in the frameworks of this method the knowledge of magnetic line configuration of the measuring RF magnetic field of the single-layer flat coil sensor is required and modeling of the distribution of magnetic field lines is urgent.

So, at measurements by the use of the SFCO method, in order to get optimal conditions for its application, it is necessary to know the magnetic field distribution near the flat coil that is the resonance circuit of the oscillator. The field distribution has been experimentally investigated in detail in [4]. In this paper we have presented the theoretical investigation of the distribution of measuring magnetic field near flat coil and have compared with available experimental data in [4].

Theory. The calculations are based on the well known formula in electrodynamics for deriving the magnetic vector potential [6]

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$$\vec{A}=\frac{1}{c}\int\frac{\vec{J}}{R}dV. \tag{1}$$

The calculations have been made in the spherical coordinate system with the origin in the center of the single-layer flat coil. It is evident from the symmetry of formula (1) in this coordinate system that only the component A_φ of the vector potential of magnetic field of the flat coil – serving as a sensor of the oscillator – differs from zero, since the vector of current in this coil is directed along this axis. This component could be written in the following form

$$A_\varphi=\frac{1}{c}\int\frac{J_\varphi}{|\vec{\Re}-\vec{R}|}R^2\sin\theta dRd\theta d\varphi, \tag{2}$$

where $\vec{\Re}$ is the radius vector of the point, where the field is being calculated, c is the speed of light, J_φ is the current per unit area of the wire, $\vec{R}$ is the radius vector of a point on the wire and the integration is realized over the entire space.

It will be assumed in what follows that the current producing the measuring RF magnetic field flows on the surface of flat coil turns and so is perpendicular to the radius vector of the point on the wire. In case of such a close approximation to the reality, it is easily obtained with the help of expression (2) that

$$A_\varphi=\frac{1}{c}\int\frac{\frac{I\cos\varphi}{2\pi}d\alpha(\vec{x}+\vec{r})_{hor}}{|\vec{\Re}-(\vec{x}+\vec{r})|}d\varphi=\frac{I}{2\pi c}\int_\alpha\int_\varphi\frac{(x-r\cos\alpha)d\alpha*\cos\varphi d\varphi}{\sqrt{\Re^2+(\vec{x}+\vec{r})^2-2\Re|\vec{x}+\vec{r}|\cos\gamma}}, \tag{3}$$

where γ is the angle between $\vec{\Re}$ and $\vec{x}+\vec{r}$. From properties of Legendre special functions follows that

$$A_\varphi=\frac{I}{2c}\int_\alpha\sum_{L=1}^{\infty}\frac{(x-r\cos\alpha)(\min(\Re,|\vec{x}+\vec{r}|))^L}{(\max(\Re,|\vec{x}+\vec{r}|))^{L+1}}\cdot\frac{2}{L(L+1)}P_L^1(\cos\theta)P_L^1(\cos\theta')d\alpha, \tag{4}$$

where θ' is the angle between the coil's axis and $\vec{R}$. Note that as

$$\cos\theta'=\frac{r\sin\alpha}{\sqrt{x^2+r^2-2rx\cos\alpha}}, \tag{5}$$

i.e. $\cos\theta'$ is a function only of α (α is the angle between the wire's radius and the turn radius). Hence, based on the above calculations, it is easy to write A_φ as

$$A_\varphi=\frac{I}{c}\sum_{L=1}^{\infty}\frac{P_L^1(\cos\theta)}{L(L+1)}Q_L, \tag{6}$$

where the following notation was used:

$$Q_L=\begin{cases}\displaystyle\int_0^{2\pi}(x-r\cos\alpha)\frac{|\vec{x}+\vec{r}|^L}{\Re^{L+1}}P_L^1\left(\frac{r\sin\alpha}{\sqrt{x^2+r^2-2xr\cos\alpha}}\right)d\alpha, & \text{if} \quad \Re\geq|\vec{x}+\vec{r}|,\\ \displaystyle\int_0^{2\pi}(x-r\cos\alpha)\frac{\Re^L}{|\vec{x}+\vec{r}|^{L+1}}P_L^1\left(\frac{r\sin\alpha}{\sqrt{x^2+r^2-2xr\cos\alpha}}\right)d\alpha, & \text{if} \quad \Re\leq|\vec{x}+\vec{r}|.\end{cases} \tag{7}$$

Now, proceeding from an expression for the vector potential of the field and using the well known relation $\vec{H} = \operatorname{rot}\vec{A}$, we come to an expression

$$\vec{H} = \frac{1}{\Re\sin\theta}\cdot\frac{\partial(A_\varphi\sin\theta)}{\partial\theta}\hat{e}_r - \frac{1}{\Re}\cdot\frac{\partial(\Re A_\varphi)}{\partial\Re}\hat{e}_\theta . \tag{8}$$

After corresponding mathematical transformations the final analytical formula for H_r is written as

$$H_r = \frac{I}{c\Re}\sum_{L=1}^{\infty} Q_L P_L(\cos\theta) \tag{9}$$

and for H_θ:

$$H_\theta = -\frac{1}{\Re}\cdot\frac{\partial(\Re A_\varphi)}{\partial\Re} = -\frac{I}{c\Re}\sum_{L=1}^{\infty}\frac{P_L^1(\cos\theta)}{L(L+1)}\cdot\frac{\partial(\Re Q_L)}{\partial\Re} . \tag{10}$$

It is easy to see that

$$\frac{\partial(\Re Q_L)}{\partial\Re} = \begin{cases} -LQ_L, & \text{if} \quad \Re \geq \sqrt{x^2+r^2-2xr\cos\alpha}, \\ (L+1)Q_L, & \text{if} \quad \Re \leq \sqrt{x^2+r^2-2xr\cos\alpha}. \end{cases} \tag{11}$$

So, the mathematical expression for H_θ transforms to

$$H_\theta = -\frac{I}{c\Re}\sum_{L=1}^{\infty}\frac{P_L^1(\cos\theta)Q_L}{L(L+1)}\cdot\begin{cases} -L, & \text{if} \quad \Re \geq \sqrt{x^2+r^2-2xr\cos\alpha}, \\ (L+1), & \text{if} \quad \Re \leq \sqrt{x^2+r^2-2xr\cos\alpha}. \end{cases} \tag{12}$$

Analytical expressions (9) and (12) have been obtained for the components of magnetic field produced by an x radius coil made from r radius wire. However, for flat single-layer coil serving as a sensor for the measuring oscillator used in our experimental research of superconductivity, we have dealt with the Archimedean spiral, which in case when the number of turns exceeded 10 may be considered to a good approximation as a system of tightly enclosed coaxial rings. In this model the calculations are significantly simplified and the magnetic field of coil is a vector sum of magnetic fields produced by constituent rings.

If we denote the total number of turns (rings) in the coil by N, the radius of the t-th turn by R_t, and Q_L of this turn correspondingly by $Q_{L,t}$, then the expressions for H_r and H_θ take the following form:

$$H_r = \sum_{t=1}^{N}\frac{I}{c\Re}\sum_{L=1}^{\infty} Q_{L,t}P_L(\cos\theta), \tag{9a}$$

$$H_\theta = -\frac{I}{c\Re}\sum_{L=1}^{\infty}\frac{P_L^1(\cos\theta)Q_{L,t}}{L(L+1)}\cdot\begin{cases} -L, & \text{if} \quad \Re \geq \sqrt{R_t^2+r^2-2R_t r\cos\alpha}, \\ (L+1), & \text{if} \quad \Re \leq \sqrt{R_t^2+r^2-2R_t r\cos\alpha}, \end{cases} \tag{12a}$$

where $\Re$ and θ are the polar coordinates of the point, where the magnetic field is calculated. Now note that $R_t = R_{hole} + (2t-1)r$, where R_{hole} is the radius of hole of the coil center, and $Q_{L,t}$ depends only on the turn number (t), radius of the wire (r) and R_{hole}.

To simplify calculations we will use the fact that as only $Q_{L,t}$ is a function of R_t, hence, the summation symbol refers only to $Q_{L,t}$ terms. Furthermore, in the case of even L-s, it is easy to show that $Q_{L,t}$ terms are zero. Also we will use the $\sum_{t=1}^{N} Q_{L,t} = Q_L$ notification (with redesignation of the former Q_L).

To get handy analytic expressions for an experimenter we need to make some approximations based on the form of coil as well as the characteristics of our superconductivity research.

At the preparation of coil it is impossible to avoid the technological hole at its center that is equal to R_{hole} =0.5 *mm* for the coil at issue in the present paper. As the coil was wound using the wire of $2r = 0.1$ *mm* diameter, so irrespective of the turn number $r / R_t \leq 0.1$. Hence, $Q_{L,t}$ is expandable into a series in small quantity of r / R_t, it being shown in our subsequent calculation that in the expansion into series in r / R_t one can confine to the 5-th term accuracy.

In our experimental research of superconductivity we have dealt with the components of measuring magnetic field strength parallel (H_x or H_y) and perpendicular (H_z) to the coil plane that are expressed through H_θ and H_r (see (9a), (12a)).

So, it is important to have analytic expressions just for H_x (or H_y) and for the H_z components. It is easy to see that

$$H_x \equiv H_y = H_r \sin\theta + H_\theta \cos\theta \tag{13}$$

and for the H_z component

$$H_z = H_r \cos\theta - H_\theta \sin\theta . \tag{14}$$

Results and Discussions. One of possible ways of testing the coil's field configuration is the calibration by moving different diameter disk-shaped copper plates towards the coil's face up to the given distance and back [4]. Having in view the comparison of calculated values of the horizontal components of field strength ($H_x \equiv H_y$) with corresponding experimental data (see Fig. 1, 2), we availed ourselves of the calibration data in [4]. Note in this connection that in this work the authors used copper plates for measurements. In the inset to Fig.1 are presented the measured in [4] data on the dependence of TD oscillator frequency on the distance of cooper plate from the coil surface (h).The strength of measuring magnetic field at every point of space can be calculated using formula (13) derived by us. Since a plate can be considered as a system of balls that are at equal distances from the coil surface, it makes sense to compare the experimental data obtained in [4] and presented in the inset to Fig. 1 with the dependence of near field value of magnetic coil on the distance from coil surface calculated according to above relation (13) for the region of field homogeneity. Fig.1. shows the graph of calculated dependence of H_x (or H_y) component of near RF magnetic field of flat coil on the real distance distance h from the coil at the length equal to half of the radius from the coil axis (along the line $\Re \sin\theta = R_{coil} / 2$). Note that for proportions of copper plates $(0.2–0.6)R_{coil}$ used for measurements in [4] the measuring RF field is still homogeneous (see the inset from the cited work to Fig. 2).

In the inset to Fig. 2. are measured data [4] on the dependence of TD oscillator frequency shift on the diameter of copper discs that are at fixed distance from the coil surface (the frequency shift is normalized by the area of the plate).

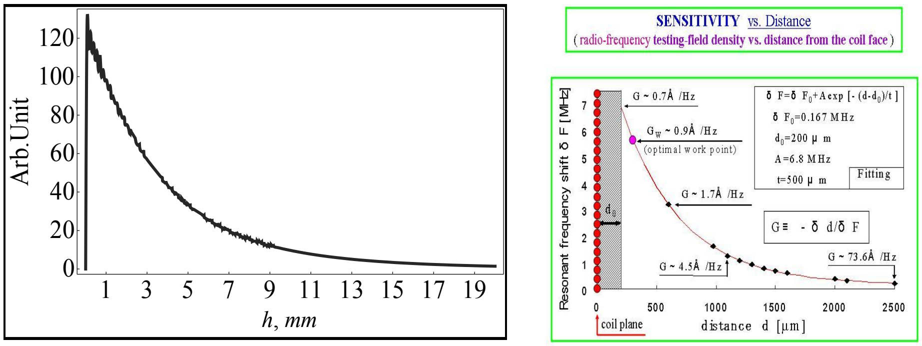

Fig. 1. H_x (or H_y, parallel to the coil face components) component vs. the distance (h) from the coil surface, obtained from formula (13). The parameters of calculations are $\Re\sin\theta = 5$ *mm*, $r = 0.05$ *mm*, $R_{hole} = 0.5$ *mm* and $N = 100$ turns, $R_{coil} = 10$ *mm*.

Inset: The oscillator frequency vs. the distance of cooper plate from the coil of Φ_{coil}~8 *mm* diameter [4].

The continuous lines connecting the points represent the abovementioned dependence for various distances of the plates from the coil. As the contribution of the inhomogeneous part of the field to the frequency shift of oscillator was shown in [4] to be small compared with that of the homogeneous part of the field, that gave us grounds for making comparison of the observational results obtained in [4] with our calculation results given in the same figure.

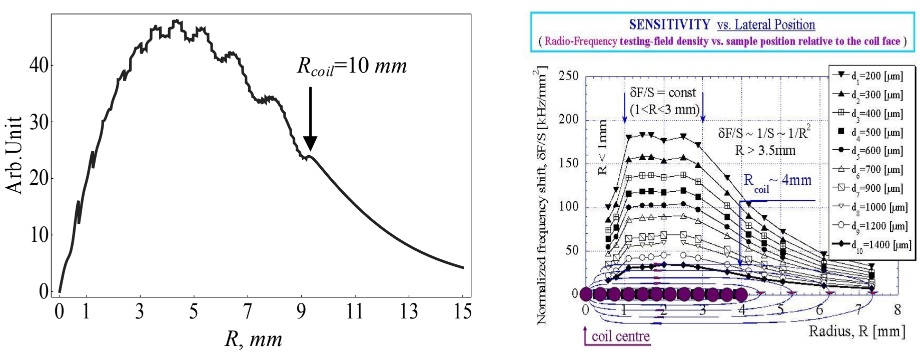

Fig. 2. The dependence of H_x (or H_y) components on the distance R from the coil axis in the plane that is at 5 *mm* distance from the coil face, obtained from formula (13). The parameters of calculations are $\Re\cos\theta = 5$ *mm*, $r = 0.05$ *mm*, $R_{hole} = 0.5$ *mm* and N=100 turns, $R_{coil} = 10$ *mm*.

Inset: The dependence of oscillator frequency shift on the distance from the coil for copper discoid plates of various radii, when the plates are brought from infinity down to the given distance [4] (the frequency shift is normalized to the plate area). Continuous lines connect points that correspond to cases, where plates are at the same distance from the coil.

Seen in Fig. 2 is the plot of calculated dependence of H_x (or H_y) component of field on the distance (R) from the coil axes in the plane that is parallel to the coil face and is at 5 *mm* distance from that (i.e. in $\Re\cos\theta$=5 *mm* plane). The qualitative coincidence of results witnesses that the partial contribution of the inhomogeneous part of field to the frequency shift of oscillator is small compared with that of homogenous part. If instead of a plate a ball-shaped object is moved parallel to the coil face, then the frequency shift dependence on the ball distance from the coil axis would not coincide with theoretical result since in the central part of field the influence of H_z component of the coil on the ball becomes substantial.

The inset to Fig. 3 shows the dependence of frequency shift of the flat coil-based oscillator measured in [7] on the distance of ball-shaped particle from the coil axis at its motion in the plane parallel to the coil surface along its diameter. The curve in Fig. 3 gives the calculated dependence of H_z component (14) on the distance R from the coil axis in the plane that is parallel to the coil face at 5 *mm* distance from that (i.e. in $\Re\cos\theta = 5$ *mm* plane). A qualitative comparison of these curves is justified, because, as was mentioned above, the presence of ball near the coil axis sufficiently distorts the H_z component of magnetic field.

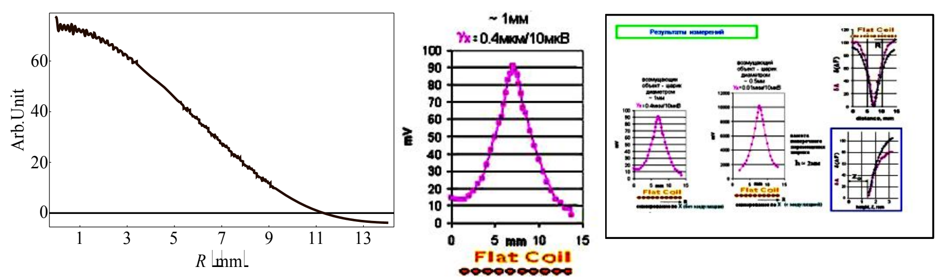

Fig. 3. The calculated dependence of H_z component on the distance R from the coil axis in the plane that is at 5 *mm* distance from the coil face, obtained by formula (14). The parameters of calculations are $\Re\cos\theta = 5$ *mm*, $r = 0.05$ *mm*, $R_{hole} = 0.5$ *mm* and $N = 100$ turns, $R_{coil} = 10$ *mm*.

Inset: The measured dependence of oscillator frequency $\delta(\Delta F)$ and amplitude δA shifts on the distance from the coil axis of ball shaped tin (Sn) particle with ~1 *mm* radius that is moved at 1 *mm* distance from the coil surface along its diameter [7].

It follows from the results, obtained that when thin plate-like objects are brought nearer to the flat coil, it would be more correct to compare the value of oscillator frequency shift with H_x (or H_y) component of the measuring field, whereas in case of translation of metallic ball-shaped objects parallel to the coil face the value of oscillator frequency shift should be compared (in the first order of approximation) with the value of H_z component of measuring field.

So, we received evidence that the results of our calculations qualitatively agree with the obtained experimental data. Hence, we infer that it is possible to model the flat coil technique used by us for research of fine peculiarities of superconductivity by theoretical methods discussed in this paper.

The deduction and investigation of the dependence expressed by formula (14) and presented in Fig. 3 was important and up to date since it enabled us to clarify the possibility of distinguishing between the magnetic and nonmagnetic regions of a superconducting unique state using the single layer flat coil as a novel type probe as was discussed in [3]. The calculations show that at definite value of R (see Fig. 3.) the value assigned to the magnetic field strength becomes negative that implied the reversal of field direction. The reversal of magnetic field is clearly seen in experiments, where the electromotive force induction by RF magnetic field of the coil is measured [8].

Conclusions. Further improvements in TD activated single-layer flat-coil-oscillator technique would create better opportunities for studying the phenomenon predicted in 1964 by Fulde and Ferrell [9] and independently by Larkin and Ovchinikov [10] (the so-called FFLO effect). The first observation of this phenomenon was qualitative [11]. However, no detailed investigation of this phenomenon has been made yet due to the lack of instrumentation with enough resolution.

In accordance with estimations made in [12–14], we hope that in case of creation of a flat coil-based sensor (microscope) using 1 *mm* coil as a probe, it would be possible to attain enough spatial resolution for investigation of the above phenomena. In this connection note, that lithographic coils of 1 *mm* in diameter are already available in market (usmicrowaves.com). With such a SFCO based technique, as an effective-probing element at hand, one may reach the aim of detection (and then, the visualization) of the node structure in FFLO-state [9–11] with a spatial resolution of about 0.1 μm. That makes the problem of theoretical and experimental study of the magnetic field configuration near the flat coil face up to date.

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REFERENCES

1. **Gevorgyan S.G., Kiss T.** et al. // Rev. Sci. Instrum., 2000, v. 71, p. 1488–1494.
2. **Gevorgyan S.G., Movsesyan G.D.** et al. // Rev. Sci. Instrum., 1998, v. 69, p. 2550–2560.
3. **Gevorgyan S., Kiss T.** et al. // Supercond. Sci. Technol., 2001, v. 14, p. 1009–1013.
4. **Gevorgyan S.G., Kiss T.** et al. // Physica C: Supercond. & Appl., 2001, v. 366, p. 6–12.
5. **Gevorgyan S.G., Kiss T.** et al. // Physica C: Supercond. & Appl., 2001, v. 363, p. 113–118.
6. **Landau L.D., Lifshiz E.M.** Field Theory. M.: Nauka, 1988, p. 142 (in Russian).
7. **Gevorgyan S.G. and Azaryan M.G.** Proc. of the 5-th International Conference on Semiconductor Micro- & Nanoelectronics. Aghveran, Armenia, 2005, p. 82–86.
8. **Gevorgyan S.G., Muradyan S.T., Kurghinyan B.K., Qerobyan M.I.** // Proceedings of the YSU. Physical and Mathematical Sciences, 2011, № 3, p. 47–51.
9. **Fulde P., Ferrell R.A.** // Phys. Rev., 1964, v. 135, № 3A, p. A550–A563.
10. **Larkin A.I., Ovchinikov Y.N.** // JETP, 1964, v. 47, №3 (9), p. 1136–1146; Sov. Phys. JETP, 1965, v. 20, p. 762.
11. **Radovan H.A., Murphy T.P., Palm E.C., Tozer S.W., Hall D. and Sarrao J.L.** // Journal of Low Temperature Physics, 2003, v. 133, p. 377–386; NHMFL Reports, 2003, v. 10 (3), № 1. magnet.fsu.edu
12. **Gevorgyan S., Shirinyan H.** et al. // Nucl. Instr. & Meth in Phy. Res. A, 2004, v. 520, p. 314–319.
13. **Karapetyan G.H., Shirinyan H.G., Gevorgyan S.G., Gevorgyan V.G.** Proc. of the VII Conf. on Educational, Scientific & Technol. Appl. in LabVIEW Environm. & Technol. of NI., 2008, November 28–29. Moscow, Russia, v. 1, № 1.0.2., p. 8–12.
14. **Gevorgyan S.G., Shirinyan H.G., Karapetyan G.H., Gevorgyan G.S., Polyanskii A.A.** // Proceedings of the YSU. Physical and Mathematical Sciences, 2009, № 2, p. 50–54.

Բ. Կ. Կուրղինյան. Հարթ կոճի շրջապատի չափող դաշտի մոդելավորումը՝ գերհաղորդական երևույթի թույլ արտահայտված առանձնահատկությունների բացահայտումն արդյունավետ դարձնելու նպատակով էջ. 53–59

Իրականացվել է ԳՀ ուսումնասիրման արդյունավետ մեթոդներից մեկի՝ թունելային դիոդով ակտիվացվող հարթ կոճով, զգայուն ինքնագեներատորի տվիչը հանդիսացող միաշերտ, հարթ կոճի շրջապատի ռադիոհաճախային (ՌՀ) չափող էլեկտրամագնիսական դաշտի բաշխվածության մոդելավորում: Արտածվել է այդ դաշտի մագնիսական բաղադրիչը հաշվելու համար անալիտիկ արտահայտություն, ինչը թույլ է տալիս կոճը շրջապատող տարածության ցանկացած կետում հաշվել չափող ՌՀ դաշտի մագնիսական բաղադրիչ-ները՝ ինչպես կոճի մակերևույթին զուգահեռ, այնպես էլ դրան ուղղա-հայաց ուղղություններով: Հաշվարկների նպատակն է, ԳՀ երևույթի թույլ արտահայտված (բայց երևույթի ընկալման իմաստով կարևոր) առանձնա-հատկությունների, հարթ կոճի մեթոդով բացահայտման ընթացքում փորձ-նականորեն չափող տեխնիկայի օպտիմալ զգայնությանը հասնելը: