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HYPERNUCLEAR MATTER IN COMPACT STARS

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We discuss the recently developed energy density functional for hypernuclear matter, which is based on simultaneous calculation of heavy single-Λ hypernuclei and compact stars containing hypernuclear core. The nucleonic matter is described in terms of a density-dependent parametrization of nucleon-meson couplings, whereas the hyperon-meson couplings are deduced from the octet model. We identify the parameter space of hyperon-meson couplings for which massive stellar configurations with $M \leq 2.1 M_\odot$ exist and at the same time the laboratory Λ-hypernuclear data can be described.

***Keywords*:** neutron stars, hyperonic matter, equation of state.

Introduction. The recent observations of two-solar-mass pulsars in binary orbits with white dwarfs [1, 2] spurred an intensive discussion of the phase structure of dense matter, which is consistent with the implied observational lower bound on the maximum mass of any sequence of compact stars based on the unique equation of state (EoS) of dense matter. Hyperons become energetically favorable once the Fermi energy of neutrons exceeds their rest mass. The onset of hyperons reduces the degeneracy pressure of a cold thermodynamic ensemble, therefore, EoS becomes softer than in absence of the hyperons (for example, [3,4]). As a result the maximum possible mass of a compact star decreases to values which contradict the observations. This contradiction is known as "hyperonization puzzle".

The current and upcoming experimental studies of the properties of Λ-hypernuclei in laboratories such as HKS experiment at JLab in the US, J-PARC experiments in Japan etc., will greatly advance our understanding of the strange sector of the nuclear forces and properties of hypernuclei.

The experimental observations of bound Λ-hypernuclei imply that the interaction must be attractive enough to bind a Λ particle to a medium and heavy mass nucleus. At the same time the existence of two-solar-mass pulsars requires sufficient repulsion (at least at high densities) to guarantee the stability of hypernuclear compact stars, if such exist. Therefore, the combined laboratory and astronomical data limit from above and below the attraction among hyperons in nuclear medium in any particular model.

In this contribution we review how these bounds can be used to constrain the relativistic density functional theory (DFT) of hypernuclear matter, as discussed originally in [5, 6], where an extension of nuclear density functional with a density-dependent parameterization of the couplings, was extended to the hypernuclear sector within the SU(3) symmetric model. The initial focus was on the sensitivity of EoS of hypernuclear matter to the unknown hyperonscalar-meson couplings [5]. Later this model was tested by carrying

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out calculations of a number of hypernuclei and by providing a *combined constraint* on the parameters of the underlying DFT by invoking both the astronomical and laboratory data on hypernuclear systems [6]. The coupling of σ-meson to the Λ-hyperon were optimized to fit the data on hypernuclei, thus narrowing down the parameter space. The parameter space of the remaining $\sigma-\Sigma$ coupling was then constrained using some general inequalities as well as astronomical observations of the $2M_{\odot}$ pulsars.

Theoretical Model and Choice of Couplings. The density functional for hypernuclear matter is based on the Lagrangian. Relativistic Lagrangian density of our model reads

$$\begin{aligned}\mathcal{L} &= \sum_B \bar{\psi}_B \left[\gamma^\mu \left(i\partial_\mu - g_{\omega B}\omega_\mu - \frac{1}{2} g_{\rho B} \boldsymbol{\tau}\cdot\boldsymbol{\rho}_\mu\right) - (m_B - g_{\sigma B}\sigma)\right]\psi_B \\ &+ \frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - \frac{m_\sigma^2}{2}\sigma^2 - \frac{1}{4}\omega^{\mu\nu}\omega_{\mu\nu} + \frac{m_\omega^2}{2}\omega^\mu\omega_\mu - \frac{1}{4}\boldsymbol{\rho}^{\mu\nu}\boldsymbol{\rho}_{\mu\nu} \\ &+ \frac{m_\rho^2}{2}\boldsymbol{\rho}^\mu\cdot\boldsymbol{\rho}_\mu + \sum_\lambda \bar{\psi}_\lambda (i\gamma^\mu\partial_\mu - m_\lambda)\psi_\lambda,\end{aligned} \tag{1}$$

where the B-sum is over the $J^P = (1/2)^+$ baryon octet, ψ_B are the baryonic Dirac fields with masses m_B. The meson fields σ, ω_μ and $\boldsymbol{\rho}_\mu$ mediate the interaction among baryon fields, $\omega_{\mu\nu}$ and $\boldsymbol{\rho}_{\mu\nu}$ represent the field strength tensors of vector mesons and m_σ, m_ω, and m_ρ are their masses. The baryon-meson coupling constants are denoted by g_{mB}. The last term of Eq. (1) stands for the contribution of the free leptons, where the λ-sum runs over the leptons e^-, μ^-, ν_e and ν_μ with masses m_λ. The density dependence of the couplings implicitly takes into account many-body correlations among nucleons which are beyond the mean-field approximation. The nucleon-meson coupling constants are parametrized as $g_{iN}(\rho_B) = g_{iN}(\rho_0)h_i(x)$ for $i = \sigma, \omega$, and $g_{\rho N}(\rho_B) = g_{\rho N}(\rho_0)\exp[-a_\rho(x-1)]$ for the $\boldsymbol{\rho}_\mu$-meson, where ρ_B is the baryon density, ρ_0 is the saturation density, $x = \rho_B/\rho_0$ and the explicit form of the functions $h_i(x)$ and the values of couplings can be found elsewhere [5]. This density functional is consistent with the following parameters of nuclear systems: saturation density $\rho_0 = 0.152\ fm^{-3}$, binding energy per nucleon $E/A = -16.14$, incompressibility $K_0 = 250.90$, symmetry energy $J = 32.30$, symmetry energy slope $L = 51.24$, symmetry incompressibility $K_{sym} = -87.19\ MeV$ all taken at saturation density. Obtained values of parameters are in an excellent agreement with the nuclear phenomenology.

The hyperon–meson couplings are fixed according to the SU(3)-flavor symmetric octet model. Due to the universal coupling of the $\boldsymbol{\rho}_\mu$ meson to the isospin current and the ideal mixing between the ω and ϕ mesons, the couplings between hyperons and vector mesons are as follows

$$x_{\rho\Xi} = 1, \quad x_{\rho\Sigma} = 2, \quad x_{\omega\Xi} = 1/3, \quad x_{\omega\Sigma} = x_{\omega\Lambda} = 2/3, \quad x_{\rho\Lambda} = 0, \tag{2}$$

where we defined the ratios $x_{\rho\Xi} = g_{\rho\Xi}/g_{\rho N}$, $x_{\rho\Sigma} = g_{\rho\Sigma}/g_{\rho N}$, etc.

Within the octet model the baryon–scalar-mesons couplings of the scalar octet can be expressed in terms of only two parameters, the nucleon–a_0 meson coupling constant g_S and the $F/(F+D)$ ratio of the scalar octet. Allowing for mixing of the scalar singlet state, the couplings of the baryons with the σ-meson obey the following relation [5] $2(g_{\sigma N} + g_{\sigma\Xi}) = 3g_{\sigma\Lambda} + g_{\sigma\Sigma}$. We assume that the hyperon coupling constants must be positive and less than the nucleon coupling constants. Solving this equation for one of the dependent hyperon–σ-meson coupling constant, say $g_{\sigma\Xi}$, one finds

$$1 \le \frac{1}{2}(3x_{\sigma\Lambda} + x_{\sigma\Sigma}) \le 2. \tag{3}$$

These inequalities define a bound on the area spanned by the coupling constants $x_{\sigma\Lambda}$ and $x_{\sigma\Sigma}$, which we will constrain further in the following.

Properties of Λ-hypernuclei $^{17}_{\Lambda}$O, $^{41}_{\Lambda}$C, *and* $^{49}_{\Lambda}$C *for the models a, b, and c. The columns list the single-particle energy of the* Λ $1s_{1/2}$ *state, the binding energy and the rms radii for neutrons, protons and Λ-hyperon.*

	Λ $1s_{1/2}$ state, *MeV*	E/A, *MeV*	r_p, *fm*	r_n, *fm*	r_Λ, *fm*
$^{17}_{\Lambda}$O					
a	0.846	−7.443	2.609	2.579	8.313
b	−4.564	−7.760	2.606	2.576	3.203
c	−27.279	−9.035	2.563	2.534	1.977
$^{41}_{\Lambda}$C					
a	0.934	−8.336	3.372	3.319	8.710
b	−8.519	−8.565	3.370	3.317	3.168
c	−35.224	−9.199	3.347	3.294	2.298
$^{49}_{\Lambda}$C					
a	0.973	−8.442	3.389	3.576	8.825
b	−9.882	−8.662	3.387	3.571	3.140
c	−37.257	−9.207	3.365	3.548	2.419

The same density functional, which is derived from the Lagrangian (1) can be applied to finite Λ-nuclei. The results of these calculations are presented in Table, where we list the single-particle energy of the Λ $1s_{1/2}$ state, the binding energy of the nucleus and the rms radii r_B for neutrons, protons and the Λ-hyperon. Table shows clearly that the single-particle energy of the Λ $1s_{1/2}$ state is very sensitive to the value of the Λ-σ coupling. The experimental data on properties of a number of Λ-hypernuclei such as the single-particle energy of the Λ $1s_{1/2}$ state, has been used to construct a mass formula, which extends the familiar Bethe–Weizsäcker mass formula to include in addition to the non-strange nuclei the Λ-hypernuclei. A comparison with the predictions of this mass formula shows that the Λ $1s_{1/2}$ states in the model *b* are too weakly bound, whereas those in the model *c* are too strongly bound. Therefore, we proceed further to fine-tune the $x_{\sigma\Lambda}$ coupling in order to fit the values of the single-particle energies, i.e. separation energies of the Λ particle, obtained from the mass formula. The *optimal model* obtained in this way has $x_{\sigma\Lambda} = 0.6164$. Within this optimal model we have recomputed the properties of $^{17}_{\Lambda}$O, $^{41}_{\Lambda}$Ca and $^{49}_{\Lambda}$Ca, which agree well with other models (see [6]).

From the study of the dependence of EoS on the variation of the Σ-σ coupling at $T = 0$ at fixed value of Λ-σ we find that the stiffest EoS is obtained for the smallest value of $x_{\sigma\Sigma} = 0.15$. EoS band generated by the values of $0.15 \leq x_{\sigma\Sigma} \leq 0.65$ is bounded from below by EoS which, as we shall see, is incompatible with the lower bound on the maximum mass of a compact star. Therefore, the parameter space can be narrowed down further by exploring the masses of corresponding stellar configurations.

Fig. 1 shows the gravitational masses (in solar units) vs radii for our sequences of stars. First, we see that large enough masses can be obtained within the parameter range covered. However, for large enough $x_{\sigma\Sigma}$ the maximum masses of the sequences drop below the observational value $2M_{\odot}$, specifically for $x_{\sigma\Lambda} = 0.6164$ this occurs for $x_{\sigma\Sigma} \geq 0.45$. The predicted radii of massive hypernuclear stars are in the range of 13 *km* and are typically larger than the radii of their purely nucleonic counterparts.

Fig. 2 shows the parameter space covered by the coupling constants $x_{\sigma\Sigma}$ and $x_{\sigma\Lambda}$. The shaded (blue online) area is the parameter space consistent with Eq. (3). The dot corresponds to the values of these parameters predicted by the Nijmegen Soft-Core (NSC) potential.

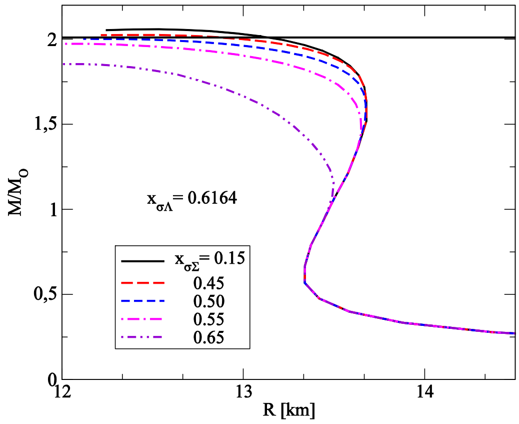

Fig. 1. The mass-radius relations for compact hypernuclear stars at zero temperature. We fix $x_{\sigma\Lambda} = 0.6164$ and assign values to $x_{\sigma\Sigma}$ from the range $0.15 \leq x_{\sigma\Sigma} \leq 0.65$ as indicated in the plot. The horizontal line shows the observational lower limit on the maximum mass $2.01(\pm 0.04)M_{\odot}$.

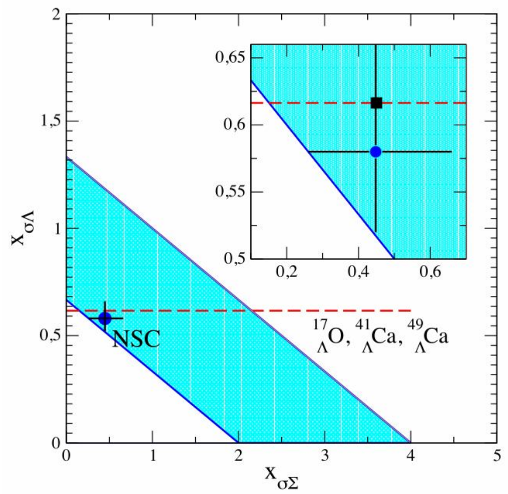

Fig. 2. The parameter space spanned by $x_{\sigma\Lambda}$ and $x_{\sigma\Sigma}$, where the inset enlarges the physically relevant area. The shaded (blue online) area corresponds to the inequality (3). The dot corresponds to the values $x_{\sigma\Lambda} = 0.58$ and $x_{\sigma\Sigma} = 0.448$ derived from the NSC potential. The dashed (red online) line shows the best fit value of $x_{\sigma\Lambda} = 0.6164$ derived from hypernuclei. The square in the inset shows the limiting value of $x_{\sigma\Sigma} = 0.45$ for fixed $x_{\sigma\Lambda} = 0.6164$ beyond, which no stars with $2M_{\odot}$ exist.

The dashed (red online) line, which is the optimal value of $x_{\sigma\Lambda}$ implied by the hypernuclear data. The solid vertical and horizontal lines show the parameter space explored in [5]. Finally, the square in the inset shows the maximal value of $x_{\sigma\Sigma} \simeq 0.45$ (at fixed $x_{\sigma\Lambda}$), which is still consistent with the $2M_{\odot}$ maximum value of a configuration. Thus, we conclude that the optimal values of the parameters correspond to

$$x_{\sigma\Lambda} = 0.6164, \qquad 0.15 \leq x_{\sigma\Sigma} \leq 0.45. \tag{4}$$

The first value is set by the study of (heavy) hypernuclei, the upper limit of the second value is set by the $2M_{\odot}$ constraint, whereas the lower limit is set by the requirement of the consistency with inequality (3).

Conclusion. Because the information on the properties of hypernuclear matter is far less extensive than for nucleons, it is currently impossible to exclude hyperons as constituents of densest regions of compact stars. Our study [5] confirms this within a specific relativistic density functional approach to hypernuclear matter with tuned hyperon–scalar-meson couplings. We find that hyperonization in massive stars is favored for small ratios of the hypernuclear-to-nuclear couplings; in particular, hyperons need to be coupled to scalar mesons weaker than predicted by the SU(6) quark model. For certain values of the hyperon–scalar meson couplings hypernuclear EoS can still produce stellar equilibrium configurations of compact stars compatible with the two-solar-mass pulsar observations.

Simultaneous fits to the medium-heavy Λ-hypernuclei and the requirement that the maximum mass of a hyperonic compact star is at least two-solar masses puts additional constraint on the Λ hyperon coupling. This allowed us to narrow down significantly the parameter space of couplings of DFT, the range of optimal values of parameters is given in Eq. (4).

While our work was carried out within a specific parameterization of the hypernuclear density functional, it provides a proof-of-principle of the method for constraining any theoretical framework that describes hypernuclear systems using current laboratory and astrophysical data.

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