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ABOUT CONTACT PROBLEMS FOR AN ELASTIC HALF-PLANE AND THE INFINITE PLATE WITH TWO FINITE ELASTIC OVERLAYS IN THE PRESENCE OF SHEAR INTERLAYERS

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The problems of contact interaction are observed for an elastic half-plane and the infinite plate, which are strengthened, along the line (in the plane) by two finite overlays (stringers) with different elastic characteristics and constant small thickness. The contact interaction between deformable foundations and overlays is realized through a shear layers (in form of glue layers) having different physical–mechanical properties and geometric configuration. The determination problem of unknown contact stresses are reduced to the systems of Fredholm's integral equations of the second kind within the different finite intervals, which in the certain region of the change of characteristic parameter typical to the problems, may be solved by the method of successive approximations. Possible particular cases are observed and the character and behavior of contact stresses are illustrated.

***Keywords*:** contact, elastic half-plane, infinite plate (sheet), overlay (stringer), shear, system of integral equation, operator equation.

In the articles [1, 2], the solutions of problems is reduced to the systems of singular integral-differential equations of the second kind with Cauchy's kernel, where its solutions are constructed using apparatus of Chebishev's orthogonal polynomials. Contact problem for the infinite plate with two finite stringers through of shear interlayer with one of stringers is observed in the article [3].

In present paper in contrast to [1, 2], a different approach to solving the problems is applied, which allows to reduce the solution of the problems to the systems of Fredholm's integral equations of the second kind.

The problem for elastic deformable base in the form of half-plane was chosen as the main. During the process of problem solving the results for the infinite plate are also presented, using the same designations where possible.

Let an elastic half-plane (with the elasticity modulus E or shear modulus G, the Poisson's ratio ν) is strengthened by two finite, small thickness (h_1, h_2) overlays on intervals $[-a, a]$ and $[b, c]$ $(b>a)$ of its boundary at $y=0$ (in xOy plane), with modulus of elasticity E_1 and E_2, when $x\in[-a, a]$ and $x\in[b, c]$ respectively. The contact interaction between the half-plane and overlays was realized through shear

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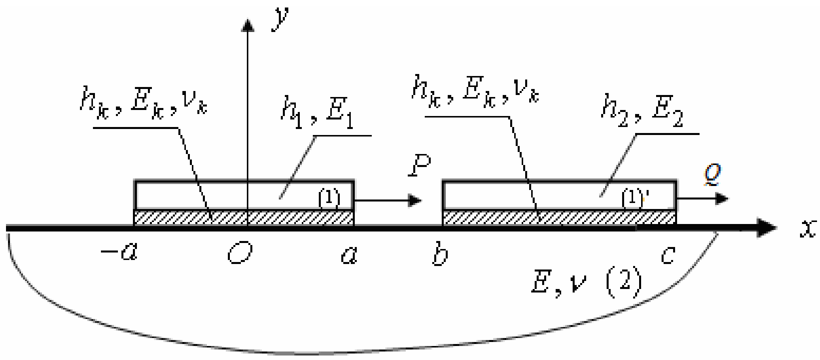
interlayers (layers of glue) with characteristics E_k, ν_k, h_k. The problem reduces to the determination of contact stresses, when horizontal forces P and Q are applied in the points of overlays $x = a$ and $x = c$ respectively, along the Ox axis (see Figure).



It is supposed that for the overlays (stringers) the model of one measurement elastic continuum with confrontation contact along the line is realized, and for the interlayers it is the pure shear condition realized, due to which only the tangential (shear) contact stresses are acting in contact parts [1–4].

Taking into account above mentioned and assumptions from [1–4], the equilibrium differential equations for the overlays on the $[-a,a]$ and $[b,c]$ intervals will be written in the following form:

$$\frac{d^2u^{(1)}(x)}{dx^2}=\frac{\tau_1(x)}{E_1h_1},\qquad -a\le x\le a,\tag{1}$$

$$\frac{d^2u_1^{(1)}(x)}{dx^2}=\frac{\tau_2(x)}{E_2h_2},\qquad b\le x\le c,\tag{2}$$

where $u^{(1)}(x)$ and $u_1^{(1)}(x)$ are the horizontal displacements of the points of the overlays, $\tau_1(x)$ and $\tau_2(x)$ are the tangential (shear) contact stresses, acting under overlays on the $[-a,a]$ and $[b,c]$ intervals correspondingly.

Now let write horizontal displacements of the boundary points of elastic half-plane $u^{(2)}(x,0)$ in the following form:

$$u^{(2)}(x,0)=\frac{1}{\pi A}\int_{-a}^{a}\left(\ln\frac{1}{|x-s|}+C\right)\tau_1(s)\,ds+\frac{1}{\pi A}\int_{b}^{c}\left(\ln\frac{1}{|x-s|}+C\right)\tau_2(s)\,ds,\tag{3}$$

where $A=E/2(1-\nu^2)=2G(1-\chi^2)$, $\chi^2=(1-2\nu)/2(1-\nu)$, C is arbitrary constant.

Assuming that each differential element of the glue layer is in the condition of pure shear [1–4], the following contact conditions are obtained:

$$u^{(1)}(x)-u^{(2)}(x,0)=k\tau_1(x),\qquad -a\le x\le a,\tag{4}$$

$$u_1^{(1)}(x)-u^{(2)}(x,0)=k\tau_2(x),\qquad b\le x\le c,\tag{5}$$

where $k=h_k/G_k$, $G_k=E_k/2(1+\nu_k)$, G_k is the shear modulus of glue material.

For the elastic infinite plate, which is defined to the conditions of generalized strain state, it is supposed that stringers defined on the surface infinite plate at line $y=0$ (xOy its average plane), in formulas (1)–(5), $\tau_j(x)$ $(j=1,2)$ must change substitution into $b_1^*\tau_j(x)$, h_j $(j=1,2)$ it is necessary to substitute on cross-sectional areas of stringers $F_j=b_1^*h_j$, and A on $A^*=4Ed/(1+\nu)(3-\nu)=8Gd/(3-\nu)$, d is the thickness plate, G is the shear modulus of material plate, b_1^* is the width stringers on the contact parts, k on $k^*=\dfrac{k}{b_1^*}$.

Further by virtue of (4) and (5), Eq.(1) and Eq.(2) can be written in the form:

$$\frac{d^2u^{(1)}}{dx^2}-\alpha_1^2u^{(1)}(x)=-\alpha_1^2u^{(2)}(x,0),\quad -a\le x\le a,\tag{6}$$

$$\frac{d^2u_1^{(1)}}{dx^2}-\alpha_2^2u_1^{(1)}(x)=-\alpha_2^2u^{(2)}\left(x,0\right),\qquad b\le x\le c,\tag{7}$$

where we have also the following boundary conditions respectively

$$\left.\frac{du^{(1)}}{dx}\right|_{x=-a}=0,\qquad \left.\frac{du^{(1)}}{dx}\right|_{x=a}=\frac{P}{E_1h_1},\tag{8}$$

$$\left.\frac{du_1^{(1)}}{dx}\right|_{x=b}=0,\qquad \left.\frac{du_1^{(1)}}{dx}\right|_{x=c}=\frac{Q}{E_2h_2}.\tag{9}$$

Here $\alpha_1^2=1/kE_1h_1$, $\alpha_2^2=1/kE_2h_2$.

For the elastic plate we must change α_j^2 on $\bar{\alpha}_j^2=b_1^*G_k/h_kE_jF_j$ $(j=1,2)$. Further, the solution of boundary value problem (6) and (8) we obtain in the form

$$u^{(1)}(x)=u_0^{(1)}(x)+\alpha_1^2\int_{-a}^{a}G(x,s)u^{(2)}(s,0)ds,\quad -a\le x\le a,\tag{10}$$

where $u_0^{(1)}(x)$ is general solution of the homogenous equation corresponding to Eq. (6) with the boundary conditions (8) and has the form

$$u_0^{(1)}(x)=\frac{P\,\mathrm{ch}\,\alpha_1(x+a)}{\alpha_1E_1h_1\mathrm{sh}\,2a\alpha_1},$$

and $u_0(x)=\alpha_1^2\int_{-a}^{a}G(x,s)u^{(2)}(s,0)ds$ is a particular solution of Eq.(6) with zero boundary conditions $\left(du^{(1)}/dx\right)_{x=-a}=0$, $\left(du^{(1)}/dx\right)_{x=a}=0$, where $G(x,s)$ is Green's function [5], and

$$G(x,s)=\frac{1}{\alpha_1\mathrm{sh}2a\alpha_1}\begin{cases}\mathrm{ch}\alpha_1(x-a)\mathrm{ch}\alpha_1(s+a), & x>s,\\ \mathrm{ch}\alpha_1(x+a)\mathrm{ch}\alpha_1(s-a), & x<s.\end{cases}$$

Function $G(x,s)=G(s,x)$ is continuous function obviously.

Similarly, the solution of boundary value problem (7), (9) we obtain in the form

$$u_1^{(1)}(x)=u_*^{(1)}(x)+\alpha_2^2\int_{b}^{c}K(x,s)u^{(2)}(s,0)ds,\quad b\le x\le c,\tag{11}$$

where $u_*(x)$ is a particular solution of Eq.(7) with zero boundary conditions corresponding to (9) and has the form

$$u_*(x)=\alpha_2^2\int_b^c K(x,s)u^{(2)}(s,0)ds,$$

where

$$K(x,s)=\frac{1}{\alpha_2\mathrm{sh}[\alpha_2(c-b)]}\begin{cases}\mathrm{ch}\alpha_2(x-c)\mathrm{ch}\alpha_2(s-b), & x>s,\\ \mathrm{ch}\alpha_2(x-b)\mathrm{ch}\alpha_2(s-c), & x<s,\end{cases}$$

$u_*^{(1)}(x)$ is general solution of the homogenous equation corresponding to Eq. (7) with boundary conditions (9) and has the form

$$u_*^{(1)}(x)=\frac{Q\mathrm{ch}[(x-b)\alpha_2]}{\alpha_2 E_2 h_2 \mathrm{sh}(c-b)\alpha_2},$$

where, $K(x,s)=K(s,x)$ is a continuous function.

Now, by virtue of (10) and (11) and according conditions (4) and (5), we obtain the following equations:

$$k\tau_1(x)+u^{(2)}(x,0)=\alpha_1^2\int_{-a}^{a}G(x,s)u^{(2)}(s,0)ds+u_0^{(1)}(x),\quad -a\le x\le a,\qquad(12)$$

$$k\tau_2(x)+u^{(2)}(x,0)=\alpha_2^2\int_b^c K(x,s)u^{(2)}(s,0)ds+u_*^{(1)}(x),\quad b\le x\le c.\qquad(13)$$

For future, it should be noted that spectrum of the symmetric second-order differential operator $D=-\frac{d^2}{dx^2}+\alpha_1^2 I$, which definition domain are twice continuous differentiating functions satisfying the boundary conditions $\left(du^{(1)}/dx\right)_{x=-a}=0$ and $\left(du^{(1)}/dx\right)_{x=a}=0$, are eigenvalues $\lambda_n=\alpha_1^2+\frac{n^2\pi^2}{4a^2}$ and corresponding them eigenfunctions are $\cos\left[\frac{n\pi(x+a)}{2a}\right]$, where $n=0,1,2,...$

Further, it is known [5], that symmetric quite continuous integral operator B:

$$B\varphi=\int_{-a}^{a}G(x,s)\varphi(s)ds,$$

which acts in the space $L_2(-a,a)$ is an inverse operator of D.

Hence, we have

$$\int_{-a}^{a}G(x,s)\cos\left[\frac{n\pi(s+a)}{2a}\right]ds=\frac{4a^2}{4a^2\alpha_1^2+n^2\pi^2}\cos\left[\frac{n\pi(x+a)}{2a}\right],\quad n=0,1,2,...,\qquad(14)$$

$$\int_b^c K(x,s)\cos\left[\frac{n\pi(s-b)}{c-b}\right]ds=\frac{(c-b)^2}{(c-b)^2\alpha_2^2+n^2\pi^2}\cos\left[\frac{n\pi(x-b)}{c-b}\right],\quad n=0,1,2,...,\qquad(15)$$

where the functions $\cos\left[\frac{n\pi(x+a)}{2a}\right]$ and $\cos\left[\frac{n\pi(x-b)}{c-b}\right]$, $n=0,1,2,...$, form full orthogonal systems in the spaces $L_2(-a,a)$ and $L_2(b,c)$ accordingly.

Now by virtue of (3), from (12) and (13) we will obtain the following system:

$$\tau_1(x)+\frac{1}{\pi kA}\left[\int_{-a}^{a}(\ln\frac{1}{|x-s|}+C)\tau_1(s)ds+\int_{b}^{c}(\ln\frac{1}{|x-s|}+C)\tau_2(s)ds\right]=$$
$$=\frac{\alpha_1^2}{\pi kA}\int_{-a}^{a}G(x,s)\left[\int_{-a}^{a}(\ln\frac{1}{|s-t|}+C)\tau_1(t)dt+\int_{b}^{c}(\ln\frac{1}{|s-t|}+C)\tau_2(t)dt\right]ds+$$
$$+u_0^{(1)}(x)/k,\qquad -a\le x\le a, \tag{16}$$
$$\tau_2(x)+\frac{1}{\pi kA}\left[\int_{-a}^{a}(\ln\frac{1}{|x-s|}+C)\tau_1(s)ds+\int_{b}^{c}(\ln\frac{1}{|x-s|}+C)\tau_2(s)ds\right]=$$
$$=\frac{\alpha_2^2}{\pi kA}\int_{b}^{c}K(x,s)\left[\int_{-a}^{a}(\ln\frac{1}{|s-t|}+C)\tau_1(t)dt+\int_{b}^{c}(\ln\frac{1}{|s-t|}+C)\tau_2(t)dt\right]ds+$$
$$+u_*^{(1)}(x)/k,\qquad b\le x\le c.$$

Further, replacing variables x, s and t with ax, as and at respectively, we will obtain

$$p(x)+\frac{\delta^2}{\pi}\int_{-1}^{1}\ln\frac{1}{|x-t|}\,p(t)dt-\frac{a\alpha_1^2\delta^2}{\pi}\int_{-1}^{1}G(ax,as)\int_{-1}^{1}\ln\frac{1}{|s-t|}p(t)dt\,ds+$$
$$+\frac{\delta^2}{\pi}\int_{\delta_1}^{\delta_2}\ln\frac{1}{|x-t|}q(t)dt-\frac{a\alpha_1^2\delta^2}{\pi}\int_{-1}^{1}G(ax,as)\int_{\delta_1}^{\delta_2}\ln\frac{1}{|s-t|}q(t)dt\,ds-\frac{u_0^{(1)}(ax)}{k}=0,$$
$$-1\le x\le 1,$$
$$q(x)+\frac{\delta^2}{\pi}\int_{-1}^{1}\ln\frac{1}{|x-t|}p(t)dt-\frac{a\alpha_1^2\delta^2}{\pi}\int_{\delta_1}^{\delta_2}K(ax,as)\int_{-1}^{1}\ln\frac{1}{|s-t|}p(t)dt\,ds+ \tag{17}$$
$$+\frac{\delta^2}{\pi}\int_{\delta_1}^{\delta_2}\ln\frac{1}{|x-t|}q(t)dt-\frac{a\alpha_1^2\delta^2}{\pi}\int_{\delta_1}^{\delta_2}K(ax,as)\int_{\delta_1}^{\delta_2}\ln\frac{1}{|s-t|}q(t)dt\,ds-\frac{u_*^{(1)}(ax)}{k}=0,$$
$$\delta_1\le x\le\delta_2,$$

since according to (14) and (15), we have also the following equalities :

$$\int_{-1}^{1}G(ax,as)ds=\frac{1}{a\alpha_1^2},\quad \int_{\delta_1}^{\delta_2}K(ax,as)ds=\frac{1}{a\alpha_2^2}. \tag{18}$$

Here $\delta^2=\dfrac{a}{kA},\quad \delta_1=\dfrac{b}{a},\ \delta_2=\dfrac{c}{a},\quad p(x)=\tau_1(ax),\quad q(x)=\tau_2(ax).$

One can represent the system of integral equations (17) in the following form:

$$p(x)+\delta^2\int_{-1}^{1}H(x,t)p(t)dt+\delta^2\int_{\delta_1}^{\delta_2}H(x,t)q(t)dt=p_0(x),\quad -1\le x\le 1,$$
$$q(x)+\delta^2\int_{\delta_1}^{\delta_2}\Pi(x,t)q(t)dt+\delta^2\int_{-1}^{1}\Pi(x,t)p(t)dt=q_0(x),\quad \delta_1\le x\le\delta_2, \tag{19}$$

where

$$H(x,t)=\frac{1}{\pi}\left(\ln\frac{1}{|x-t|}-a\alpha_1^2\int_{-1}^{1}G(ax,as)\ln\frac{1}{|s-t|}ds\right),$$

$$\Pi(x,t)=\frac{1}{\pi}\left(\ln\frac{1}{|x-t|}-a\alpha_2^2\int_{\delta_1}^{\delta_2}K(ax,as)\ln\frac{1}{|s-t|}ds\right), \qquad (20)$$

$$p_0(x)=\frac{u_0^{(1)}(ax)}{k}=\frac{PG_k\,\mathrm{ch}[\alpha_1 a(x+1)]}{\alpha_1 E_1 h_1 h_k\,\mathrm{sh}2a\alpha_1},\quad q_0(x)=\frac{u_*^{(1)}(ax)}{k}=\frac{Q\,G_k\mathrm{ch}[\alpha_2 a(x-\delta_1)]}{\alpha_2 E_2 h_2 h_k\,\mathrm{sh}[a\alpha_2(\delta_2-\delta_1)]}.$$

For the elastic plate (sheet) in the system (19) we should replace δ^2 on $\bar{\delta}^2=ab_1^*(1+\nu)(3-\nu)/4k\,dE$ and $p_0(x)=b_1^*u_0^{(1)}(ax)/k$, $q_0(x)=b_1^*u_*^{(1)}(ax)/k$.

Note that the system (17) or (19) are obtained from (16) by the change of integration order, the validity of which yields from Fubini's theorem [5]. This theorem is used often in future without special mentioning.

Now let consider several possible particular cases, which can be directly obtained from the system of integral equations (19). In the case $\delta^2=0$, we obtain the solution of considered problem for the case of a rigid foundation (i.e. when $E\to\infty$) in the form $p(x)=p_0(x)$ and $q(x)=q_0(x)$ respectively. In the case of one finite overlay, which is given on the interval $[-a,\ a]$ (or on the interval $[b,\ c]$ respectively), instead of system (19) we will have the Fredholm's integral equation of the second kind with respect to unknown function $p(x)$ (or with respect to unknown function $q(x)$ on the interval $[\delta_1,\ \delta_2]$ respectively). Note that the system (19) was obtained without using the equilibrium conditions of overlays:

$$\int_{-1}^{1}p(x)\,dx=P/a,\quad \int_{\delta_1}^{\delta_2}q(x)\,dx=Q/a. \qquad (21)$$

In system (19) the conditions (21) are satisfied automatically, since the following equalities take place: $\int_{-1}^{1}p_0(x)\,dx=P/a$, $\int_{\delta_1}^{\delta_2}q_0(x)\,dx=Q/a$.

One can easily verify this, integrating the first equation of the system (19) from -1 to 1, and the second one from δ_1 to δ_2, and then changing the order of integration in obtained repeated integrals and taking into account the equalities $\int_{-1}^{1}H(x,t)\,dx=0$, $\int_{\delta_1}^{\delta_2}\Pi(x,t)\,dx=0$, which following from (18).

Thus, the solution of the problem is reduced to the solution of the system of Fredholm's integral equation of the second kind with the kernels, which are square integrable by two variables, and with right parts of which are the solutions of the problem in the case of rigid foundation. It is easy to see from the system (19), that in the ends of overlays $x=\pm1$ and $x=\delta_1$, $x=\delta_2$, the values of unknown contact stresses $p(x)$ and $q(x)$ are finite.

Further, let write the system (19) in the following form:

$$\varphi+K\varphi=g_0, \qquad (22)$$

where

$$\varphi = \begin{pmatrix} p \\ q \end{pmatrix}, \ g_0 = \begin{pmatrix} p_0 \\ q_0 \end{pmatrix}, \ K = \begin{pmatrix} \delta^2 k_{11} & \delta^2 k_{12} \\ \delta^2 k_{21} & \delta^2 k_{22} \end{pmatrix},$$

$$k_{11}p = \int_{-1}^{1} H(x,t)p(t)dt, \ k_{12}q = \int_{\delta_1}^{\delta_2} H(x,t)q(t)dt,$$
$$k_{21}p = \int_{-1}^{1} \Pi(x,t)p(t)dt, \ k_{22}q = \int_{\delta_1}^{\delta_2} \Pi(x,t)q(t)dt, \tag{23}$$

Now let's consider operator Eq.(22) in Banach space B by meaning of vector-function $X = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$, where $X_1 \in L_2(-1, 1)$, $X_2 \in L_2(\delta_1, \delta_2)$ and with the norm

$$\|X\| = \max\left\{\|X_1\|_{L_2(-1,1)}, \|X_2\|_{L_2(\delta_1,\delta_2)}\right\}.$$

L_2 is a space of square integrable functions.

Operators k_{11}, k_{22} are acting in the spaces $L_2(-1, 1)$, $L_2(\delta_1, \delta_2)$ respectively and operators k_{12} and k_{21} are acting in the following form: $k_{12} : L_2(\delta_1,\delta_2) \to L_2(-1,1)$, $k_{21} : L_2(-1,1) \to L_2(\delta_1,\delta_2)$.

Obviously, the operator K acts in the B space and is Fredholm's operator. Then operational Eq. (22) in the B space can be solved by the method of successive approximations, if $\|K\| < 1$, where

$$\|K\| = \max\left\{\delta^2\left(\|k_{11}\| + \|k_{12}\|\right), \delta^2\left(\|k_{21}\| + \|k_{22}\|\right)\right\}.$$

Therefore, condition $\|K\| < 1$ will be realized, if

$$\delta^2\left(\|k_{11}\| + \|k_{12}\|\right) < 1, \ \delta^2\left(\|k_{21}\| + \|k_{22}\|\right) < 1. \tag{24}$$

Then the solution of Eq. (22) will be written in the form:

$$\varphi = (I + K)^{-1} g_0 = \sum_{n=0}^{\infty} (-1)^n K^n g_0.$$

Now let determine the values of δ^2 parameter for which the conditions (24) will be satisfied. From (23), by virtue of Cauchy–Bunyakovski inequality, we will get:

$$\|k_{11}\| \le c_1, \ c_1 = \left(\int_{-1}^{1}\int_{-1}^{1} H^2(x,t)\,dxdt\right)^{1/2}, \quad \|k_{12}\| \le c_2, \ c_2 = \left(\int_{\delta_1}^{\delta_2}\int_{-1}^{1} H^2(x,t)\,dxdt\right)^{1/2},$$
$$\|k_{21}\| \le c_3, \ c_3 = \left(\int_{-1}^{1}\int_{\delta_1}^{\delta_2} \Pi^2(x,t)\,dxdt\right)^{1/2}, \quad \|k_{22}\| \le c_4, \ c_4 = \left(\int_{\delta_1}^{\delta_2}\int_{\delta_1}^{\delta_2} \Pi^2(x,t)\,dxdt\right)^{1/2}. \tag{25}$$

Obviously, the expressions for c_i $(i = 1,2,3,4)$ are hard to count, but they can be estimated. It was found out that the following estimates take place:

$$c_1 < \left(\int_{-1}^{1}\int_{-1}^{1} \ln^2 |x-t| \, dx\,dt \right)^{1/2}, \qquad c_2 < \left(\int_{\delta_1}^{\delta_2}\int_{-1}^{1} \ln^2 |x-t| \, dx\,dt \right)^{1/2},$$
$$c_3 < \frac{l}{2}\left(\int_{-1}^{1}\int_{\delta_1}^{\delta_2} \ln^2 |x-t| \, dxdt \right)^{1/2}, \quad c_4 < \frac{l}{2}\left(\int_{\delta_1}^{\delta_2}\int_{\delta_1}^{\delta_2} \ln^2 |x-t| \, dx\,dt \right)^{1/2}, \; l=\delta_2-\delta_1. \tag{26}$$

For receiving the estimates (26), we will consider c_1: from Eq. (14) it is obvious that $\cos\left[\dfrac{m\pi(x+1)}{2}\right]$ $(m=0,1,2,...)$ is complete orthogonal system in $L_2(-1,1)$. Then, according to Parseval's equality we will have,

$$\int_{-1}^{1} H^2(x,t)\,dx = \sum_{m=1}^{\infty} H_m^2(t), \quad -1<t<1,$$

where

$$H_m(t) = \int_{-1}^{1} H(x,t)\cos\left[\frac{m\pi(x+1)}{2}\right] dx, \quad m=1,2,...,$$

since, we have that $H_0=0$. Further, according to (14), we have

$$H_m(t) = \left(1-\frac{4\alpha_1^2}{4\alpha_1^2+m^2\pi^2}\right) C_m(t), \quad m=1,2,...,$$

where $C_m(t) = \int_{-1}^{1} \ln\dfrac{1}{|x-t|}\cos\left[\dfrac{m\pi(x+1)}{2}\right] dx, \qquad m=1,2,...,$

therefore, $\int_{-1}^{1} H^2(x,t)\,dx = \sum_{m=1}^{\infty}\left(1-\dfrac{4\alpha_1^2}{4\alpha_1^2+m^2\pi^2}\right)^2 C_m^2(t) < \sum_{m=1}^{\infty} C_m^2(t), \quad -1<t<1.$

On the other hand, in virtue Cauchy–Bunyakovski inequality, we obtain

$$\sum_{m=1}^{\infty} C_m^2(t) \le \int_{-1}^{1} \ln^2 |x-t| dx.$$

Therefore, $c_1 < \left(\int_{-1}^{1}\int_{-1}^{1} \ln^2 |x-t| \, dx\,dt \right)^{1/2}$. The rest of estimates (26) are obtained similarly.

Then the conditions (24) will be realized, if

$$\delta^2 < (c_1+c_2)^{-1} = c', \; \delta^2 < (c_3+c_4)^{-1} = c''.$$

Therefore, the conditions of realizations (24) are obtained in the form: $\delta^2 < \min(c', c'')$, where c', c'' are positive numbers less then unity.

Now, since $\delta_1 = b/a$, $\delta_2 = c/a$, and accepting $b = 2a$, $c = 4a$, after counting (26) the following estimations are obtained:

$$c_1 < \left(\int_{-1}^{1}\int_{-1}^{1} \ln^2 |x-t| \, dx\,dt \right)^{1/2} \approx 2.76, \quad c_2 < \left(\int_{2}^{4}\int_{-1}^{1} \ln^2 |x-t| \, dx\,dt \right)^{1/2} \approx 2.20,$$
$$c_4 < \left(\int_{2}^{4}\int_{2}^{4} \ln^2 |x-t| \, dx\,dt \right)^{1/2} \approx 2.76, \quad c_3 < \left(\int_{-1}^{1}\int_{2}^{4} \ln^2 |x-t| \, dx\,dt \right)^{1/2} \approx 2.20. \tag{27}$$

Therefore, conditions of realization (24) will be obtained in the form $\delta^2 < 0.20$.

The values of contact stresses $p(x)$ and $q(x)$ in the points $x = \pm 1$ and $x = \delta_1$, $x = \delta_2$, of overlays we obtain from (19), substituting $x = \pm 1$ and $x = \delta_1$, $x = \delta_2$ accordingly.

Further, note that the posed problems may be interpreted as a contact problem with piecewise homogenous finite overlay, which somehow is separated from deformable foundation in the part $x \in (a,b)$, if supposing that force P is unknown (inner force) and an identical, but opposite force is applied in the point $x = b$ of the overlay, which is defined by the condition $\int_b^c \tau_2(s)ds = Q - P$. In this case, in the parts connected to different parts of piecewise homogenous overlay and in the points of the applied forces, for unknown share contact stresses also obtain finite values [6].

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