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M a t h e m a t i c s

SPP REAL-TIME AND DAY-AHEAD ENERGY MARKETS

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Southwest Power Pool (SPP) is a regional power transmission organization in the United States. Virtual transactions on the SPP markets are sold at auction. There are Real-Time (RT) and Day-Ahead (DA) markets, where you can buy electricity by winning an auction. Virtual transactions are a valuable component of the bilateral market. All market participants have DA and RT series of prices in hourly basis. In this article, we take a large amount of data of DA and RT prices and try to see which distribution function have the returns of RT prices. Then we make a regression between DA and RT prices, to see how they are related to each other.

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Introduction. Founded as an 11-member tight power pool in 1941, Southwest Power Pool (SPP) achieved RTO status in 2004, ensuring reliable power supplies, adequate transmission infrastructure, and competitive wholesale electricity prices for its members [1]. Virtual transactions in SPP are bids and offers submitted to take financial positions in the Day-Ahead (DA) market without the intent of delivering or consuming physical power in the Real-Time (RT) market.

All participants of the market have DA and RT hourly prices. In this paper, we take a big series of DA and RT hourly prices and try to see, what distribution function do the returns of RT prices have. Then we do the regression between DA and RT prices to see how they are connected. Then we take a part of series of RT prices, and using our regression coefficients, we predict the other part of RT prices and compare with given series of RT prices, to see if we can take part in the auction and bid a right price to win the auction, having DA prices.

Real-Time Markets. In order to explore the RT markets, we have taken RT prices of several large points in SPP markets, that sell energy, at an hourly basis, and

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we have made an index from their average that shows the behavior of the RT prices in that area. In total, these data are 34983.

Now let us take the prices of our index and remove from them extreme values, that are about 1000. Because of we have 4 years of data, there can be extreme values that have been dependent on something at a given moment and do not have a repetitive character, but can spoil the distribution function of the prices.

After all let us take the returns of the prices of index, and by the R programming language, see what distribution function will fit them.

The distribution function of Pearson Type VI is

$$F(x) = I_{(x-\gamma)/(x-\gamma+\beta)}(\alpha_1, \alpha_2), \tag{1}$$

the probability density function is

$$f(x) = \frac{\big((x-\gamma)/\beta\big)^{\alpha_1 - 1}}{\beta B(\alpha_1, \alpha_2)\big(1 + (x-\gamma)/\beta\big)^{\alpha_1+\alpha_2}}, \tag{2}$$

where B is the Beta Function, and I_z is the Regularized Incomplete Beta Function. Denote parameters: shape parameters $\alpha_1,\ \alpha_2 > 0$; scale parameter β; location parameter γ [2].

Let us fit Pearson distribution on our data of returns, and R programming language is showing, that our series of returns has Pearson Type VI distribution with $\alpha_1 = 1.43$, $\alpha_2 = 8.05$, $\gamma = 1.28$, $\beta = -1.60$ parameters.

Using parameters and Eqs. (1) and (2), we will have such form of density and distribution function for the returns of energy market RT prices:

$$f(x) = \frac{((x-1.28)/(-1.60))^{1.43-1}}{(-1.60)B(1.43,\ 8.05)(1+(x-1.28)/(-1.60))^{1.43+8.05}} =$$

$$= \frac{62.5(x-1.28)^{0.43}}{B(1.43,\ 8.05)(x-2.88)^{9.48}},$$

$$F(x) = I_{\frac{(x-1.28)}{(x-1.28-1.60)}}(1.43,\ 8.05).$$

So, we can formulate a theorem about RT prices of SPP energy markets.

Theorem. The returns of RT prices of SPP energy markets have Pearson Type VI distribution with $\alpha_2 = 8.05$, $\gamma = 1.28$, $\beta = -1.60$ parameters. Thus, distribution and density functions have following forms:

$$F(x) = I_{\frac{(x-1.28)}{(x-1.28-1.60)}}(1.43,\ 8.05), \qquad f(x) = \frac{62.5(x-1.28)^{0.43}}{B(1.43,\ 8.05)(x-2.88)^{9.48}}.$$

Day-Ahead Markets. In this section, we want to check the dependency of DA and RT prices, and see whether we can make our predictions based on DA prices or not. To check this, we will take a part of the RT prices, and do the regression DA and RT prices. Then the other part of the RT prices from regression results will be get. At the end the received prices will be compared with real prices, to check whether our method has produced a good result or not .

Table 1

Summary of the regression

Residuals				
min	1Q	median	3Q	max
–371.80	–4.79	–1.27	1.02	1231.15
coefficients				
estimate	Std. error	*t* value	Pr(>| *t* |)	
2.35670	0.40560	5.81	6.33E–0.9	***
0.86172	0.01346	64.04	<2E–16	***
residual Std. error: 24.51 on 19998 degrees of freedom				
multiple R-squared		adjusted R-squared		
0.1702		0.1701		
F-statistic: 4101 on 1 and 19998 DF.			p-value	< 2.2E–16

Let us define: a is 2/3 part and a_0 is 1/3 part of given DA prices; b is 2/3 part and b_0 is 1/3 part of RT prices.

Now let us do the regression between b and a ($b = \alpha + \beta a + \varepsilon$).

Tab. 1 shows the summary of the regression, and here we can see the coefficients, and as the p-value is much less than 0.05 we reject the null hypothesis that $\beta = 0$. Hence there is a significant relationship between the variables in the linear regression model of the data set faithful.

So we can now use our significant coefficients to forecast other 1/3 part of the RT prices. Let us define: p is our predictions of 1/3 part of the RT prices, d is difference of real prices and our predictions: $p = \hat{\alpha} + \hat{\beta} \cdot a_0, \quad d = b_0 - p$.

Our data of difference has normal distribution with parameters that are shown in Tab. 2.

Table 2

Fitting of the distribution Normal by maximum likelihood method

Parameters		
	estimate	Std. error
mean	–1.098458	0.1483830
sd	18.162835	0.1049227

The result is good, the mean of difference is -1, so the series we get is very close to the real RT prices, so we can say that having DA prices we can add the coefficients from our regression and make a better forecast of RT prices.

Conclusion. In this work we explored SPP energy markets. We made the index from RT prices of different points of energy markets to see what distribution have the returns of that prices. We have seen that in this sphere the returns of prices have Pearson Type VI distribution.

In second part we have taken DA prices and have tried to analyze that prices.

Here we found some relations between DA and RT prices, after doing regression between them. And using that relations we have done our own predictions and compare with prices. One can see that the difference of RT prices and our predictions are small enough. We have got a good result, it means that having DA prices we can use our coefficients and get a price to bid in energy market auctions and win.

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REFERENCES

1. Electric Power Markets: Southwest Power Pool (SPP). Federal Energy Regulatory Commission.
   https://www.ferc.gov/market-oversight/mkt-electric/spp.asp
2. Mathwave Data Analisys & Simulation. Pearson Type 6 Distribution.
   http://www.mathwave.com/help/easyfit/html/analyses/distributions/pearson6.html