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THE NON-CLASSICAL PROBLEM OF AN ELASTICALLY CLAMPED ORTHOTROPIC BEAM OF VARIABLE THICKNESS UNDER THE COMBINED ACTION OF COMPRESSIVE FORCES AND TRANSVERSE LOAD

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On the basis of the refined theory of orthotropic plates of variable thickness, the equations of the beam bending problem are obtained with the simultaneous action of compressive forces and transverse load. It is accepted that the edges of the beam have an elastically clamped support and the reduction of the compressive force by the support due to friction is taking into account. Passing to dimensionless quantities, a certain problem is solved. The stability of a beam is discussed. Based on the results obtained, conclusions are drawn.

MSC2010: 74K20.

***Keywords*:** orthotropic plates of variable thickness, clamped supports.

Introduction. In modern structures and devices there are often cases when thin-walled elements of variable thickness are simultaneously acted upon by transverse and longitudinal loads. The questions of the stress-strain state and the stability of such elements are investigated in several papers within the framework of the classical theory of mechanics (f.e. [1, 2]).

The use of modern materials has led to the need for carrying out the mentioned studies with refined theories, which take into account the influence of those factors that are neglected in classical theory.

In the present investigation an attempt of filling this gap is done.

Theoretical Part. Consider an orthotropic beam of length l, constant width b and variable thickness h in the right-hand Cartesian coordinate system. The main directions of the anisotropy of the material are parallel to the coordinate axes. The ends of the beam of small length $2a$ and constant thickness h_0 are inserted into the elastic array, forming an elastically clamped support [3–12]. At the ends of the beam compressive forces of the axial direction P act and the beam simultaneously carries a uniformly distributed transverse load of constant surface intensity q (Fig. 1).

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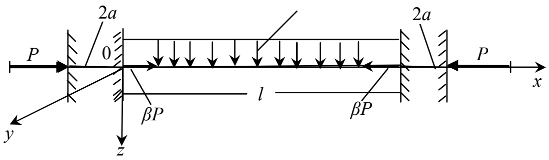

Fig. 1.

By using the refined theory of orthotropic plates of variable thickness [13], we obtain the following differential equations for the bending problem of the beam under consideration:

$$\begin{cases} \left(Eh^2\dfrac{d^2h}{dx^2}+\dfrac{12}{b}\beta P\right)\cdot\dfrac{d^2w}{dx^2}-h\left(8+Ea_{55}h\dfrac{d^2h}{dx^2}\right)\cdot\dfrac{d\varphi_1}{dx}-16\dfrac{dh}{dx}\varphi_1=12q, \\ Eh^2\dfrac{d^3w}{dx^3}+2Eh\dfrac{dh}{dx}\cdot\dfrac{d^2w}{dx^2}-Ea_{55}h^2\dfrac{d^2\varphi_1}{dx^2}-2Ea_{55}h\dfrac{dh}{dx}\cdot\dfrac{d\varphi_1}{dx}+8\varphi_1=0. \end{cases} \tag{1}$$

Here, because of the lack of voltage σ_y and neglecting of the voltage σ_z, the material parameter B_{11} is replaced by the Young modulus of the material of the axial voltage E. The number a_{55} is a known constant of elasticity material [14], connecting the deformation of transverse shift e_{xz} and tangential voltage τ_{xz}. The quantity w is the deflection, φ_1 is a function characterizing the distribution of the tangential voltage τ_{xz} in the middle plane of the beam $z=0$. The coefficient β takes into account the decrease in the compressive force P, which occurs as a result of the friction of the inserted part of the beam with the elastic mass.

The coefficient β varies within the limits $0\le\beta\le1$, which depends on the rigidity of the elastically clumped support. The case $\beta=0$ corresponds to an absolutely rigid support. Then the self-support takes all the external force and the pinch force does not act on the rod. As the parameter β increases, the support weakens, as a result of which the part of the external force acting on the rod proportionately increases. The case $\beta=1$ corresponds to the lack of support. Then all the force P applied acts on the rod. Therefore, by generalizing the above mentioned, we can say that the compressive force βP acts on the rod.

Note that in the expression of the load term Z_2 [14], the intensity of the lateral load is added to the intensity of the transverse load q that results from the compression of the curved beam.

We use the following dimensionless notation:

$$\begin{gathered} x=l\bar{x},\ \ h=h_0H,\ \ h_0=m_1l,\ \ b=m_2l,\ \ w=h_0\bar{w},\ \ Ea_{55}=\chi, \\ \varphi_1=E\bar{\varphi},\ \ q=E\bar{q}m_1^3,\ \ P=Eh_0^2\bar{P},\ \ N_x=Eh_0^2\bar{N}_x,\ \ M_x=Eh_0^3\bar{M}_x. \end{gathered} \tag{2}$$

The parameter χ takes into account the effect of the transverse shift e_{xz}. When neglecting this influence, we must set $\chi=0$. Eq. (1) in the notation of (2) take the following form:

$$\begin{cases} m_1^2\left(m_1m_2H^2\dfrac{d^2H}{d\bar{x}^2}+12\beta\bar{P}\right)\dfrac{d^2\bar{w}}{d\bar{x}^2}-m_2H\left(8+\chi m_1^2H\dfrac{d^2H}{d\bar{x}^2}\right)\dfrac{d\bar{\varphi}}{d\bar{x}}- \\ \qquad\qquad\qquad -16m_2\dfrac{dH}{d\bar{x}}\bar{\varphi}=12m_2\bar{q}m_1^2, \\ m_1^3H^2\dfrac{d^3\bar{w}}{d\bar{x}^3}+2m_1^3H\dfrac{dH}{d\bar{x}}\cdot\dfrac{d^2\bar{w}}{d\bar{x}^2}-\chi m_1^2H^2\dfrac{d^2\bar{\varphi}}{d\bar{x}^2}-2\chi m_1^2H\dfrac{dH}{d\bar{x}}\cdot\dfrac{d\bar{\varphi}}{d\bar{x}}+8\bar{\varphi}=0. \end{cases} \tag{3}$$

Let us mention that in order to obtain expressions for the lateral force N_x of the beam and the bending moment M_x we must multiply the corresponding expressions of the plate [13] by the width of the beam b. Thus for the dimensionless transverse force $\bar{N}_x$ and the bending moment $\bar{M}_x$, taking into account the notation (2), we obtain:

$$\begin{aligned} \bar{N}_x &= \frac{m_2 H}{12 m_1}\left[8\bar{\varphi} - m_1^2 H \frac{dH}{d\bar{x}}\left(m_1 \frac{d^2\bar{w}}{d\bar{x}^2} - \chi \frac{d\bar{\varphi}}{d\bar{x}}\right)\right], \\ \bar{M}_x &= -\frac{m_2 H^3}{12}\left(m_1 \frac{d^2\bar{w}}{d\bar{x}^2} - \chi \frac{d\bar{\varphi}}{d\bar{x}}\right). \end{aligned} \tag{4}$$

In the presence of an axial compressive force, the elastic pinching condition of the beam has the form [6]:

$$w = \left(a - \beta B P\right)\frac{dw}{dx} + B N_x, \quad \frac{dw}{dx} = D\left(a N_x - M_x + \beta a P \frac{dw}{dx}\right). \tag{5}$$

The parameters of the support are connected by the relation [9] $D = \dfrac{3B}{a^2}$.

In the notation $a = m_3 l,\ B = \dfrac{\bar{B}}{El}$ at $\bar{x} = 0$ and $\bar{x} = 1$ the boundary conditions of problem (5) take the form:

$$\begin{aligned} &12\bar{w} - 12\left(m_3 - \beta m_1^2 \bar{B}\bar{P}\right)\frac{d\bar{w}}{d\bar{x}} - \\ &\qquad - m_2 \bar{B} H\left[8\bar{\varphi} - m_1^2 H\left(m_1 \frac{d^2\bar{w}}{d\bar{x}^2} - \chi \frac{d\bar{\varphi}}{d\bar{x}}\right)\frac{dH}{d\bar{x}}\right] = 0, \\ &4m_3\left(m_3 - 3\beta \bar{B} m_1^2 \bar{P}\right)\frac{d\bar{w}}{d\bar{x}} - \\ &\quad - m_2 \bar{B} H\left[8 m_3 \bar{\varphi} + m_1^2 H\left(H - m_3 \frac{dH}{d\bar{x}}\right)\left(m_1 \frac{d^2\bar{w}}{d\bar{x}^2} - \chi \frac{d\bar{\varphi}}{d\bar{x}}\right)\right] = 0. \end{aligned} \tag{6}$$

Thus, the solution of the nonclassical bending problem for an elastically clamped orthotropic beam of variable thickness with the combined effect of compressive axial forces and a distributed transverse load was reduced to solving a system of fourth-order differential Eqs. (3) with boundary conditions (6).

Let us mention that when $\bar{q} = 0$ we obtain the stability problem of the beam under consideration.

Computational Part. Consider the case when the thickness of the beam varies linearly

$$h = h_0 + h_1 x \implies H = 1 + \gamma \bar{x}, \quad \gamma = \frac{h_1}{m_1}. \tag{7}$$

In this case the Eqs. (3) and the boundary conditions (6) take the form:

$$\left\{ \begin{aligned} &3m_1^2 \beta \bar{P} \frac{d^2\bar{w}}{d\bar{x}^2} - 2m_2\left(1 + \gamma\bar{x}\right)\frac{d\bar{\varphi}}{d\bar{x}} - 4m_2 \gamma \bar{\varphi} = 3m_2 \bar{q} m_1^2, \\ &m_1^3 \left(1 + \gamma\bar{x}\right)^2 \frac{d^3\bar{w}}{d\bar{x}^3} + 2m_1^3 \gamma (1 + \gamma\bar{x})\frac{d^2\bar{w}}{d\bar{x}^2} - \\ &\qquad - \chi m_1^2 (1 + \gamma\bar{x})^2 \frac{d^2\bar{\varphi}}{d\bar{x}^2} - 2\chi m_1^2 \gamma (1 + \gamma\bar{x})\frac{d\bar{\varphi}}{d\bar{x}} + 8\bar{\varphi} = 0, \end{aligned} \right. \tag{8}$$

at $\bar{x} = 0$ and $\bar{x} = 1$.

$$\begin{cases} 12\bar{w} - 12\left(m_3 - \beta m_1^2 \bar{B}\bar{P}\right)\dfrac{d\bar{w}}{d\bar{x}} - \\ \quad -m_2\bar{B}(1+\gamma\bar{x})\left[8\bar{\varphi} - m_1^2\gamma(1+\gamma\bar{x})\left(m_1\dfrac{d^2\bar{w}}{d\bar{x}^2} - \chi\dfrac{d\bar{\varphi}}{d\bar{x}}\right)\right] = 0, \\ 4m_3\left(m_3 - 3\beta\bar{B}m_1^2\bar{P}\right)\dfrac{d\bar{w}}{d\bar{x}} - m_2\bar{B}(1+\gamma\bar{x})\times \\ \quad \times\left[8m_3\bar{\varphi} + m_1^2(1+\gamma\bar{x})(1+\gamma(\bar{x}-m_3))\left(m_1\dfrac{d^2\bar{w}}{d\bar{x}^2} - \chi\dfrac{d\bar{\varphi}}{d\bar{x}}\right)\right] = 0. \end{cases} \tag{9}$$

Expressions of the dimensionless shear force and bending moment will be:

$$\begin{aligned} \bar{N}_x &= \frac{m_2(1+\gamma\bar{x})}{12m_1}\left[8\bar{\varphi} - m_1^2\gamma(1+\gamma\bar{x})\left(m_1\frac{d^2\bar{w}}{d\bar{x}^2} - \chi\frac{d\bar{\varphi}}{d\bar{x}}\right)\right], \\ \bar{M}_x &= -\frac{m_2(1+\gamma\bar{x})^3}{12}\left(m_1\frac{d^2\bar{w}}{d\bar{x}^2} - \chi\frac{d\bar{\varphi}}{d\bar{x}}\right). \end{aligned} \tag{10}$$

It is convenient to solve the problem by collocation method [15]. To this end, the unknown functions $\bar{w}$ and $\bar{\varphi}$ can be represented in the form of polynomials

$$\bar{w} = a_0 + \sum_{i=1}^{n} a_i\bar{x}^i, \qquad \bar{\varphi} = b_0 + \sum_{i=1}^{n} b_i\bar{x}^i. \tag{11}$$

We divide the interval $0 \le \bar{x} \le 1$ into n parts. Having satisfied the Eqs. (8) at the points of division and the boundary conditions (9), with respect to unknown constants a_0, a_i and b_0, b_i, we obtain a system $2(n+1)$ of linear algebraic equations with constant coefficients. By solving this system, we define the values of the functions $\bar{w}$ and $\bar{\varphi}$. The calculations will be repeated with increasing number of divisions before the practical convergence of the computation process.

Table 1

For β =0.25, 0.5, 0.75, 1.00 when $\bar{B}$=0.5, γ=1, $\bar{P}$=0.04, $\bar{q}$=0.01, $m_1 = m_3$ =0.1, m_2=0.3

χ	$\bar{x} \rightarrow$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	$\bar{w}$	0.049	0.294	0.675	0.959	1.067	0.001	0.803	0.538	0.273	0.079	0.022
0	$\bar{N}_x$	0.855	0.112	-0.07	-0.05	0.014	0.052	0.047	-0.00	-0.09	-0.20	-0.34
0	$\bar{M}_x$	-0.15	-0.04	0.047	0.100	0.122	0.112	0.070	-0.00	-0.10	-0.24	-0.39
5	$\bar{w}$	0.048	0.297	0.682	0.968	1.075	1.006	0.805	0.537	0.272	0.081	0.030
5	$\bar{N}_x$	1.062	0.229	-0.02	-0.03	0.007	0.032	0.021	-0.03	-0.12	-0.23	-0.36
5	$\bar{M}_x$	-0.19	-0.07	0.010	0.063	0.084	0.074	0.032	-0.04	-0.14	-0.27	-0.44
10	$\bar{w}$	0.047	0.301	0.691	0.980	1.086	1.013	0.809	0.538	0.272	0.083	0.037
10	$\bar{N}_x$	1.304	0.356	0.043	-0.01	0.002	0.015	-0.00	-0.06	-0.14	-0.25	-0.38
10	$\bar{M}_x$	-0.22	-0.11	-0.03	0.025	0.046	0.035	-0.01	-0.08	-0.18	-0.31	-0.48
0	$\bar{w}$	0.049	0.303	0.697	0.992	1.104	1.034	0.828	0.553	0.280	0.082	0.025
0	$\bar{N}_x$	0.870	0.123	-0.07	-0.05	0.016	0.056	0.050	-0.00	-0.09	-0.20	-0.34
0	$\bar{M}_x$	-0.15	-0.04	0.048	0.105	0.127	0.116	0.072	-0.00	-0.11	-0.24	-0.40

5	$\bar{w}$	0.049	0.313	0.721	1.024	1.135	1.059	0.845	0.562	0.284	0.085	0.033
5	$\bar{N}_x$	1.092	0.245	-0.02	-0.04	0.008	0.036	0.023	-0.03	-0.12	-0.23	-0.36
5	$\bar{M}_x$	-0.19	-0.08	0.012	0.068	0.090	0.078	0.034	-0.04	-0.15	-0.28	-0.44
10	$\bar{w}$	0.050	0.325	0.749	1.060	1.171	1.089	0.865	0.573	0.289	0.088	0.040
10	$\bar{N}_x$	1.339	0.378	0.043	-0.02	0.003	0.017	-0.00	-0.06	-0.15	-0.26	-0.39
10	$\bar{M}_x$	-0.23	-0.11	-0.03	0.030	0.052	0.040	-0.01	-0.08	-0.19	-0.32	-0.48
0	$\bar{w}$	0.049	0.311	0.721	1.028	1.143	1.069	0.854	0.569	0.288	0.085	0.028
0	$\bar{N}_x$	0.886	0.135	-0.079	-0.06	0.018	0.062	0.053	-0.01	-0.10	-0.21	-0.33
0	$\bar{M}_x$	-0.16	-0.04	0.050	0.109	0.132	0.120	0.073	-0.01	-0.11	-0.25	-0.41
5	$\bar{w}$	0.051	0.331	0.766	1.087	1.203	1.119	0.890	0.590	0.297	0.089	0.357
5	$\bar{N}_x$	1.126	0.263	-0.02	-0.04	0.010	0.408	0.025	-0.04	-0.13	-0.24	-0.37
5	$\bar{M}_x$	-0.20	-0.08	0.014	0.073	0.096	0.083	0.035	-0.04	-0.15	-0.29	-0.45
10	$\bar{w}$	0.052	0.353	0.818	1.155	1.271	1.177	0.930	0.613	0.308	0.094	0.044
10	$\bar{N}_x$	1.381	0.405	0.041	-0.02	0.004	0.021	-0.00	-0.07	-0.16	-0.27	-0.39
10	$\bar{M}_x$	-0.23	-0.12	-0.02	0.036	0.058	0.045	-0.00	-0.08	-0.19	-0.33	-0.49
0	$\bar{w}$	0.049	0.321	0.747	1.067	1.185	1.107	0.883	0.586	0.296	0.088	0.031
0	$\bar{N}_x$	0.901	0.148	-0.09	-0.06	0.020	0.068	0.057	-0.01	-0.11	-0.22	-0.33
0	$\bar{M}_x$	-0.17	-0.04	0.052	0.114	0.138	0.124	0.075	-0.01	-0.12	-0.26	-0.43
5	$\bar{w}$	0.052	0.351	0.817	1.159	1.280	1.187	0.941	0.621	0.312	0.094	0.039
5	$\bar{N}_x$	1.165	0.285	-0.03	-0.05	0.012	0.047	0.028	-0.04	-0.14	-0.25	-0.37
5	$\bar{M}_x$	-0.20	-0.08	0.016	0.079	0.102	0.088	0.037	-0.05	-0.16	-0.30	-0.46
10	$\bar{w}$	0.055	0.388	0.901	1.270	1.391	1.281	1.007	0.660	0.330	0.101	0.048
10	$\bar{N}_x$	1.431	0.439	0.039	-0.03	0.006	0.026	-0.00	-0.07	-0.17	-0.29	-0.40
10	$\bar{M}_x$	-0.24	-0.12	-0.02	0.042	0.066	0.050	-0.00	-0.09	-0.20	-0.34	-0.50

Table 2

For $\bar{B}$=0.5, γ=1, χ=8, $\bar{q}$=0.02, β=0.75, $m_1 = m_3$=0.1, m_2=0.3

$\bar{P}$	$\bar{x} \rightarrow$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	$\bar{w}$	0.092	0.562	1.288	1.827	2.032	1.904	1.527	1.021	0.519	0.157	0.064
0	$\bar{q}'$	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
0	$\bar{N}_x$	4.744	1.166	0.085	-0.08	0.004	0.068	0.017	-0.17	-0.49	-0.9	-1.43
0	$\bar{M}_x$	-0.41	-0.19	-0.03	0.073	0.114	0.094	0.015	-0.12	-0.32	-0.58	-0.90
0.02	$\bar{w}$	0.095	0.598	1.375	1.950	2.163	2.020	1.615	1.075	0.545	0.165	0.069
0.02	$\bar{q}'$	-2.46	-0.44	0.473	0.771	0.744	0.553	0.289	0.002	-0.28	-0.55	-0.78
0.02	$\bar{N}_x$	4.861	1.232	0.079	-0.094	0.006	0.079	0.021	-0.18	-0.51	-0.94	-1.44
0.02	$\bar{M}_x$	-0.42	-0.19	-0.03	0.081	0.123	0.102	0.018	-0.13	-0.33	-0.60	-0.92
0.04	$\bar{w}$	0.099	0.639	1.475	2.090	2.312	2.153	1.714	1.137	0.574	0.174	0.075
0.04	$\bar{q}'$	-5.34	-0.96	1.012	1.649	1.576	1.151	0.576	-0.04	-0.64	-1.19	-1.67
0.04	$\bar{N}_x$	4.995	1.307	0.071	-0.11	0.009	0.091	0.026	-0.20	-0.54	-0.97	-0.46
0.04	$\bar{M}_x$	-0.43	-0.20	-0.02	0.090	0.133	0.110	0.020	-0.13	-0.34	-0.61	-0.93

Table 3

For β =0.25, 0.5, 0.75, 1.00 when $\bar{B}$=0.5, γ=1, $m_1 = m_3$=0.1, $m_2 = 0.3$

$\chi \rightarrow$	0	10	0	10	0	10	0	10
$\bar{P}_{cr}$	1.198	0.552	0.599	0.276	0.399	0.184	0.299	0.138
$\bar{x}_{\max}$	0.380	0.310	0.380	0.310	0.380	0.310	0.380	0.310

Table 4

For χ=0, 10 when $\bar{B}$=0.5, β=0.5, $\bar{q}$=0.01, γ=1, $m_1 = m_3$=0.1, m_2=0.3

	$\bar{P}/\bar{P}_{cr}$	0	0.2	0.4	0.6	0.8	1
0.599	$\bar{w}_{\max}$	1.033	1.280	1.690	2.502	4.890	180.860
0.599	$\bar{x}_{\max}$	0.410	0.410	0.400	0.400	0.390	0.380
0.276	$\bar{w}_{\max}$	1.013	1.246	1.628	2.383	4.596	84.609
0.276	$\bar{x}_{\max}$	0.410	0.400	0.390	0.380	0.360	0.310

Based on the results of calculations, Tables of dimensionless calculated quantities are compiled and for clarity graphs are constructed.

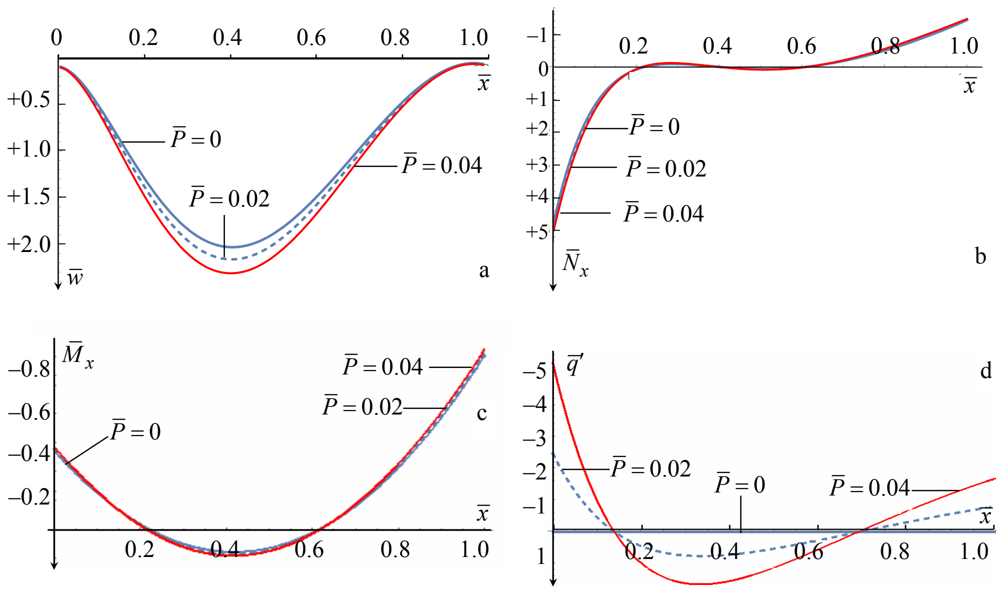

Fig. 2. $\bar{B}=0.5$, $\gamma=1.0$, $\bar{q}=0.02$, $\bar{q}'=\bar{q}-\bar{P}\cdot\bar{w}''$, $m_1=m_3=0.1$, $m_2=0.3$, $\beta=0.5$, $\chi=8.0$.

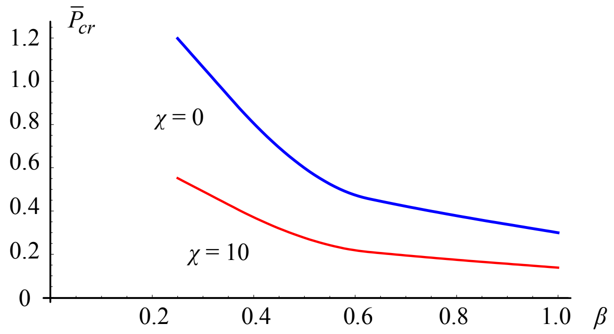

Fig. 3.

Conclusion.

1. It can be seen from Tab. 2 and Fig. 2, d, that the intensity of the transverse load $\bar{q}'$, which is the sum of the intensities of the applied load $\bar{q}$ and the load produced by the compression of the curved beam $-\bar{P}d^2\bar{w}/d\bar{x}^2$ near the elastically clamped supports, is negative, and in the rest of the beam, positive. It is easy to get involved that this is a consequence of the opposite signs of $d^2\bar{w}/d\bar{x}^2$.

2. As the parameter β increases, the elastically clamped support weakens as a result of which the part of the external compressive force acting on the beam proportionately increases. Therefore, as β increases the critical value of the external force $\bar{P}_{cr}$ decreases. The increase in the parameter χ corresponds to a decrease in the resistance to deformation of the transverse shear, which, as is known [6], leads to a decrease in the critical force. The above circumstances are demonstrated by the data of Tab. 3 and the graphs of Fig. 3.

3. With increasing the ratio $\bar{P}/\bar{P}_{cr}$, the value of the compressive forces acting on the beam approaches the critical value, which leads to an increase in deflections of the beam. When this ratio tends to one, even with an insignificant intensity of the lateral load $\bar{q}$, the maximum $\bar{w}_{\max}$ deflection of the beam sharply increases. This, of course, is a consequence of the geometrically linear statement of the problem and the assumption of unlimited elasticity of the material. In fact, a sharp increase in deflections, as a rule, is accompanied by the appearance of plastic deformations, which usually leads to the destruction of the beam. The coordinate of the maximum deflection section $\bar{x}_{\max}$ always is less than 0.5, i.e. the section of the maximum deflection is always located to the left of the middle of the span of the beam. With increasing $\bar{P}/\bar{P}_{cr}$ the section of the maximum deflection is slightly shifted towards the thin edge $\bar{x} = 0$ of the beam. These conclusions follow directly from the data in Tab. 4.

4. It can be seen from Tab. 2 and Fig. 2, that the intensity of the transverse load $\bar{q}'$ near the elastically clamped supports has a negative sign, and in the rest of the beam it is positive. It is not hard to see that this is a consequence of the opposite sign of $d^2\bar{w}/d\bar{x}^2$. For the same reason, the bending moment near the elastically-clamped supports also has a negative sign, while in the rest of the beam it is positive (Fig. 3).

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