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M a t h e m a t i c s

ON THE UNIFORM CONVERGENCE OF DOUBLE FOURIER–WALSH SERIES

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In this paper a universal function $U \in L^1[0,1)^2$, which with respect to the double Walsh system has universal property in the sense of modification, is constructed.

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Introduction. The problems of the existence of so called "universal functions" and the "universal series" are classical, and there is an extensive literature on the theory of functions, which are universal in different senses. The first example is due to Birkhoff [1], who proved the existence of an entire function $f(z)$ with the property that for an arbitrary entire function $g(z)$ there exists a subsequence $\{n_k\}_{k=1}^{\infty}$ of the natural numbers N, such that $\{f(z+n_k)\}_{k=1}^{\infty}$ converges to $g(z)$, compactly on C. Hence the sequence $\{f(z+n)\}_{n=1}^{\infty}$ of "additive translates" is dense in the space of all entire functions endowed with the topology of compact convergence.

In [2] MacLane proved a similar result for another type of universality, namely, that there exists an entire function $f(z)$, which is universal with respect to derivatives; that is, for every entire function $g(z)$ and for each number $r>0$, there exists a increasing sequence of natural numbers $\{n_k\}_{k=1}^{\infty}$, so that the sequence $\{f^{(n_k)}(z)\}_{k=1}^{\infty}$ uniformly converges to $g(z)$ on $|z| \le r$.

In [3] Marcinkiewicz proved that for any nonzero null sequence $h_n \to 0$ there exists a continuous function $F \in C[0,1]$ F: $[0,1] \to \mathcal{R}$ having the property: for any measurable function $f(x):[0,1]$ there is a subsequence $n_k \nearrow^{\infty}$ such that almost everywhere on $[0,1]$

$$\frac{F(x+h_{n_k})-F(x)}{h_{n_k}} \to g(x), \quad k \to \infty.$$

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In [4] Grosse–Erdmann proved the existence of an infinitely differentiable function with universal Taylor expansion. Namely, there exists a function $g(x) \in C^\infty(\mathcal{R})$, with $g(0) = 0$ such that the Taylor series at $x_0 = 0$ is locally uniformly universal in $C(\mathcal{R})$, that is for any function $f(x) \in C(R)$ with $f(0) = 0$ and a number $r > 0$ there exists a subsequence

$$S_{n_k}(g,0) = \sum_{m=1}^{n_k} \frac{g^{(m)}(0)}{m!} x^m$$

of partial sums of the Taylor series for $g(x)$, which converges to $f(x)$ uniformly on the interval $(-r,r)$.

In [5–13] we constructed functions, whose universality is manifested through Fourier series with respect to the classical systems. Here we present results having a direct relation to the present work, in [5] it is proved

Theorem 1. *There exists a (universal) function $U \in L^1[0,1)$ with strictly decreasing Fourier–Walsh coefficients such that for every almost everywhere finite measurable function on $[0,1]$ one can find a (modified) function $g \in L^\infty[0,1)$, $mes\{x \in [0,1); g \neq f\} < \varepsilon$ such that $|c_k(g)| = c_k(U), \forall k \in spec(g)$, and the Fourier–Walsh series of $g(x)$ converges uniformly on $[0,1)$, where $c_k(f) = \int_0^1 f(x)\varphi_k(x)dx$ and $spec(f) = \{k \in N, c_k(f) \neq 0\}$.*

In this paper we consider the following question: whether it is possible to get the similar result in two-dimensional case.

Note that a number of one dimensional classical results (theorems such as the L. Carleson theorem [14]: Fourier series of any function $f \in L^2[0,2\pi)$ in the trigonometric system converges almost everywhere on $[0,2\pi)$; the M. Riesz theorem [15]: Fourier series of any function $f \in L^p[0,2\pi)$, $p > 1$, in the trigonometric system converges in $L^p[0,2\pi)$ norm; the A. M. Kolmogorov theorem [16]: Trigonometric Fourier series of any function $f \in L^p[0,2\pi)$ converges in $L^p[0,2\pi)$, $p \in (0,1)$, metric) cannot be extended to two-dimensional case. In this respect, even different (spherical, rectangular, square, etc.) partial sums differ sharply from each other in their properties in matters like convergence in $L^p[0,2\pi)$, $p \geq 1$, and convergence almost everywhere. For example, in [17,18] Fefferman proved:

1) for each $p \neq 2$ there exists a function $f(x,y)$ from $L^p[0,2\pi)$, for which the spherical partial sums of the trigonometric Fourier series do not converge in $L^p[0,2\pi)$ norm;

2) there exists a continuous function $f(x,y)$, whose rectangular partial sums of the double trigonometric Fourier series diverge at any point $[0,2\pi)^2$. In [19] Grigoryan constructed a function, whose spherical partial sums of the trigonometric double Fourier series diverge in $L^p[0,2\pi)$ metrics for any $p \in (0,1)$. In paper [20] Harris constructed a function $f \in L^p[0,1)$, $1 \leq p < 2$, such that the spherical partial sums of its Fourier series in the double Walsh system diverges almost everywhere and in $L^p[0,1)$ norm. Note also that the almost everywhere convergence of the spherical

partial sums of double Fourier and Fourier–Walsh series of continuous functions is still unknown.

Let $|E|$ be the Lebesgue measure of a measurable set $E \subset [0,1)$ or $E \subset [0,1)^2$. By $L^\infty[0,1)$ we denote the space of all bounded measurable functions on $[0,1)^2$ with the norm $||\cdot||_\infty = ||\cdot||_{L^\infty[0,1)^2} = \sup\limits_{(x,y)\in[0,1)^2} \{|\cdot|\}$.

Let $\Phi = \{\varphi_k(x)\}$ be the Walsh system in the Payley ordering (see [21, 22]). We denote by $c_{k,s}(f)$ the Fourier coefficients in the double Walsh system and by $S_{N,M}((x,y),f)$ the rectangular partial sum of the Fourier–Walsh series of a function $f(x,y) \in L^p[0,1)^2$,

$$c_{k,s}(f) = \int_0^1\int_0^1 f(x,y)\varphi_k(x)\varphi_s(y)dx,\ S_{N,M}((x,y),f) = \sum_{ks=0}^{N,M} c_{k,s}(f)\varphi_k(x)\varphi_s(y).$$

The spectrum of $f(x,y)$ (denoted by $spec(f)$) is the support of $c_{k,s}(f)$, i.e. the set of the pairs of integers, where $c_{k,s}(f)$ is non-zero.

In this paper we will proof the following Theorem.

Theorem 2. *There exists a (universal) function $U \in L^1[0,1)^2$ such that for every almost everywhere finite measurable on $[0,1)^2$ function $f(x,y)$ one can find a (modified) function $\tilde{g}(x,y) \in L^\infty[0,1)^2, mes\{(x,y)\in[0,1)^2; g \neq f\} < \varepsilon$ with $|c_{k,s}(g)| = c_{k,s}(U), \forall(k,s) \in spec(g)$, whose Fourier–Walsh double series converges uniformly on $[0,1)^2$ by rectangles.*

We note that these universal functions are interesting in light of well-known classical theorems of Luzin [23] and Menshov [24] conserning the "correction of functions".

The following problems remain open:

Question 1. Is it possible to choose a modified function $g(x)$ in the Theorem 1 that $g \in C[0,1)$?

Question 2. Is the Theorems 1 and 2 true for the trigonometric system?

Main Lemmas. In the paper we use the following lemma, previously proved in [5].

Lemma A. *Let a dyadic interval Δ and numbers $m_0 \in N, \gamma \neq 0, \delta \in (0,1)$, $\theta \in \left(0, \frac{|\gamma|}{\delta}\right)$, $0 < \theta < \frac{|\gamma|}{\delta}$, be given.*

Then there exists a function $g(x)$, a measurable set $E \subset \Delta$ with $|E| > (1-\delta)|\Delta|$ and polynomials H(x), $Q(x)$ in the Walsh system $\{\varphi_k\}$ of the form

$$H(x) = \sum_{k=2^{m_0-1}}^{2^m} b_k\varphi_k(x), Q(x) = \sum_{k=2^{m_0}}^{2^m-1} \varepsilon_k b_k \varphi_k(x), \varepsilon_k = 0, \pm1, \forall k \in [2^{m_0}, 2^m),$$

which satisfy the conditions:

1) $\displaystyle\int_0^1 |H(x)|dx < \theta$;

2) $g(x) = \begin{cases} \gamma, & \text{if } x \in E, \\ 0, & \text{if } x \notin \Delta; \end{cases}$

3) $\left\| g(x) - Q(x) \right\|_{\infty} < \theta$;

4) $\max\limits_{2^{m_0-1} \le n < 2^m} \left\| \sum\limits_{k=2^{m_0}}^{n} \varepsilon_k b_k \varphi_k(x) \right\|_{\infty} < \frac{3|\gamma|}{\delta}$;

5) $\left\| g(x) - Q(x) \right\|_{\infty} < \theta$, $\max\limits_{2^{m_0-1} \le n < 2^m} \left\| \sum\limits_{k=2^{m_0}}^{n} \varepsilon_k b_k \varphi_k(x) \right\|_{\infty} < \frac{3|\gamma|}{\delta}$.

Using this Lemma and an argument of [25], we obtain the following lemma.

Lemma B. *Let number $N_0 > 1$, $\theta, \delta \in (0,1)$ and let $f(x,y)$ be a polynomial in the Walsh double system $\{\varphi_k(x)\varphi_s(y)\}$. Then there exists a function $g(x,y)$, a measurable set $E \subset [0,1)^2$ with $|E| > (1-\delta)$ and polynomials $H(x,y)$, $Q(x,y)$ in the Walsh double system $\{\varphi_k(x)\varphi_s(y)\}$ of the following form*

$$H(x,y) = \sum_{k,s=N_0}^{N} b_{k,s} \varphi_k(x)\varphi_s(y),\ Q(x,y) = \sum_{k,s=N_0}^{N} \varepsilon_{k,s}, b_{k,s} \varphi_k(x)\varphi_s(y),\ \varepsilon_{k,s} = 0; \pm 1,$$

which satisfy the conditions:

1) $\int\limits_0^1 \int\limits_0^1 |H(x,y)| dx < \theta$;

2) $g(x,y) = f(x,y), \quad for \quad all \quad (x,y) \in E$;

3) $\left\| g(x,y) - Q(x,y) \right\|_{L^\infty[0,1)^2} < \theta$;

4) $\left\| g \right\|_{L^\infty[0,1)^2} < \frac{16\|f\|_{L^\infty[0,1)^2}}{\delta^2}$.

5) $\max\limits_{N_0 \le n < N} \left\| \sum\limits_{k,s=N_0}^{n} \varepsilon_{k,s} b_{k,s} \varphi_k(x)\varphi_s(y) \right\|_{L^\infty[0,1)^2} < \frac{15\|f\|_{L^\infty[0,1)^2}}{\delta^2}$.

Proof of Theorem 2. Numbering all Walsh polynomials with rational coefficients, we can represent them as a sequence

$$\{f_n(x,y)\}_{n=1}^{\infty}. \tag{1}$$

Consecutively, applying Lemma B, one can find a sequence of functions $\{g_n^{(j)}(x,y)\}_{j=1}^{n}, n \ge 1$, sets $\{E_n^{(j)}\}_{j=1}^{n}, n \ge 1$, and polynomials of the form

$$H_n^{(j)}(x,y) = \sum_{k,s=M_n^{(j-1)}}^{M_n^{(j)}-1} b_{k,s} \varphi_k(x)\varphi_s(y), 1 \le j \le n, \tag{2}$$

$$Q_n^{(j)}(x,y) = \sum_{k,s=M_n^{(j-1)}}^{M_n^{(j)}-1} \varepsilon_{k,s}^{(n,j)} b_{k,s} \varphi_k(x)\varphi_s(y), \left(\varepsilon_{k,s}^{(n,j)} = \pm 1; 0\right), 1 \le j \le n, \tag{3}$$

where

$$\begin{gathered} 1 < M_1^{(0)} < M_1^{(1)} = M_2^{(0)} < M_2^{(1)} < M_2^{(2)} < \\ < M_{n-1}^{(n-1)} = M_n^{(0)} < M_n^{(1)} < .. < M_n^{(n)} = M_{n+1}^{(0)} < M_{n+1}^{(1)} \dots, \end{gathered} \tag{4}$$

which satisfy the conditions

$$g_n^{(j)}(x,y) = f_n(x,y) \ \text{ for } \ (x,y) \in E_n^{(j)}, |E_n^{(j)}| = 1 - 2^{-j-n}, \tag{5}$$

$$||g_n^{(j)}||_{L^\infty[0,1)^2} < 2^{2j+4}||f_n||_{L^\infty[0,1)^2},\ ||g_n^{(j)} - Q_n^{(j)}||_{L^\infty[0,1)^2} < 2^{-4n},\ 1 \le j \le n, \quad (6)$$

$$\max_{M_n^{(j-1)} \le l,m < M_n^{(j)}} \left\| \sum_{k=2^{m_n^{(j-1)}}}^{l,m} \varepsilon_k^{(n,j)} b_k \varphi_k(x)\varphi_s(y) \right\|_{L^\infty[0,1)^2} < 2^{2j+8}||f_n||_{L^\infty[0,1)^2}, \quad (7)$$

$$\int_0^1 \int_0^1 \left| H_n^{(j)}(x,y) \right| dxdy < 4^{-(n+j)}, 1 \le j \le n. \quad (8)$$

We define the function $U(x,y)$ in following way:

$$U(x,y) = \sum_{n=1}^{\infty} \sum_{j=1}^{n} \left(\sum_{k,s=M_n^{(j-1)}}^{M_n^{(j)}-1} b_{k,s}\varphi_k(x)\varphi_s(y) \right) = \sum_{k,s=0}^{\infty} b_{k,s}\varphi_k(x)\varphi_s(y). \quad (9)$$

It is clear that

$$\int_0^1 \int_0^1 |U(x,y)| dxdy \le \sum_{n=1}^{\infty} \sum_{j=1}^{n} \left(\int_0^1 \int_0^1 \left| H_n^{(j)}(x,y) \right| dxdy \right) < \sum_{n=1}^{\infty} \sum_{j=1}^{n} 4^{-(n+j)} < 1. \quad (10)$$

From this and (8), (9) we have

$$b_{k,s} = c_{k,s}(U),\ \ k,s = 0,1,2,\dots \quad (11)$$

Let $f(x,y)$ be an almost everywhere finite measurable function on $[0,1)^2$. Taking into account Luzin's theorem, without loss of generality, one may assume that $f(x,y) \in C[0,1)^2$. It is easy to see, that one can choose a subsequence $\{f_{k_n}(x,y)\}_{n=1}^{\infty}$ from the sequence (1) such that

$$\lim_{N\to\infty} \left\| \sum_{n=1}^{N} f_{k_n} - f \right\|_{L^\infty[0,1)^2} = 0,\ \ ||f_{k_n}||_{L^\infty[0,1)^2} \le 4^{-2n}, \quad n \ge 2,\ k_1 > j_0 = [\log_{\frac{1}{2}} \delta] + 1. \quad (12)$$

We put

$$Q_1(x,y) = Q_{k_1}^{(j_0+1)}(x,y),\ \ E_1 = E_{k_1}^{(j_0+1)},\ \ g_1(x,y) = g_{k_1}^{(j_0+1)}(x,y). \quad (13)$$

Suppose that natural numbers $k_1 = \nu_1 < \dots < \nu_{q-1}$, functions $f_{\nu_n}(x,y)$, $g_n(x,y),\ 1 \le n \le q-1$, sets $E_n,\ 1 \le n \le q-1$, and polynomials

$$Q_n(x,y) = Q_{\nu_n}^{(n+j_0)}(x,y) = \sum_{k,s=M_{\nu_n}^{(n+j_0-1)}}^{M_{\nu_n}^{(n+j_0)}-1} \delta_{k,s}^{(\nu_n,n+j_0)} b_{k,s}\varphi_k(x)\varphi_s(y)$$

are already defined and for all $1 \le n \le q-1$, and they satisfy the conditions:

$$\begin{gathered} g_n(x,y) = f_{k_n}(x,y),\ (x,y) \in E_n,\ |E_n| > 1 - \delta 2^{-n}, \\ \|g_n\|_{L^\infty[0,1)^2} < 5\delta^{-2}2^{-(n-8)},\ \left\| \sum_{k=1}^{q-1} [Q_k - g_k] \right\|_{L^\infty[0,1)^2} < 2^{-4(q-1)}, \\ \max_{M_{\nu_n}^{(n+j_0-1)} \le l,\ m < M_{\nu_n}^{(n+j_0)}} \left\| \sum_{k,s=M_{\nu_n}^{(n+j_0-1)}}^{l,m} \varepsilon_{k,s}^{(\nu_n,\ n+j_0)} b_{k,s}\varphi_k(x)\varphi_s(y) \right\|_{L^\infty[0,1)^2} < 2^{-n}. \end{gathered} \quad (14)$$

It is easy to see that one can choose a function $f_{\nu_q}(x,y)$ $(\nu_q > \nu_{q-1})$ from the sequence (1) such that

$$\left\| f_{\nu_q} - \left(f_{k_q} - \sum_{i=1}^{q-1} [Q_i - g_i] \right) \right\|_{L^\infty[0,1)^2} < 2^{-4q}. \tag{15}$$

By virtue of (12), (14) and (15), we have

$$\begin{aligned} \|f_{\nu_q}\|_\infty \le \left\| f_{\nu_q} - \left(f_{k_q} - \sum_{i=1}^{q-1} [Q_i - g_i] \right) \right\|_{L^\infty[0,1)^2} + \\ + \|f_{k_q}\|_{L^\infty[0,1)^2} + \left\| \sum_{i=1}^{q-1} [Q_i - g_i] \right\|_{L^\infty[0,1)^2} < 2^{-4(q-3)}. \end{aligned} \tag{16}$$

We put

$$g_q(x,y) = f_{k_q}(x,y) + [g_{\nu_q}^{(q+j_0)}(x,y) - f_{\nu_q}(x,y)], \;\; E_q = E_{\nu_q}^{(q+j_0)}, \tag{17}$$

$$Q_q(x,y) = Q_{\nu_q}^{(q+j_0)}(x,y) = \sum_{k,s=M_{\nu_q}^{(q+j_0-1)}}^{M_{\nu_q}^{(q+j_0)}-1} \varepsilon_{k,s}^{(\nu_q,q+j_0)} b_{k,s} \varphi_k(x) \varphi_s(y). \tag{18}$$

Taking into account (5) and (17), we get

$$g_q(x,y) = f_{k_q}(x,y), \;\; (x,y) \in E_q. \tag{19}$$

By virtue of (6) and (14) (15), (17) and (18) we obtain

$$\begin{aligned} \left\| \sum_{j=1}^{q} [Q_j - g_j] \right\|_{L^\infty[0,1)^2} = \left\| \sum_{j=1}^{q-1} [Q_j - g_j] + Q_q - g_q \right\|_{L^\infty[0,1)^2} \le \\ \le \left\| f_{\nu_q} - \left(f_{k_q} - \sum_{j=1}^{q-1} [Q_j - g_j] \right) \right\|_{L^\infty[0,1)^2} + \\ + ||g_{\nu_q}^{(q+j_0)} - Q_{\nu_q}^{(q+j_0)}||_{L^\infty[0,1)^2} < 2^{-4q}. \end{aligned} \tag{20}$$

Obviously (see (7) and (16)),

$$\max_{M_{\nu_q}^{(q+j_0-1)} \le l,m < M_{\nu_q}^{(q+j_0)}} \left\| \sum_{k,s=M_{\nu_q}^{(q+j_0-1)}}^{l,m} \varepsilon_{k,s}^{(\nu_q,q+j_0)} b_{k,s} \varphi_k(x) \varphi_s(y) \right\|_{L^\infty[0,1)^2} < 2^{-q}. \tag{21}$$

From (6), (14)–(16) it follows that

$$\begin{aligned} \|g_q\|_{L^\infty[0,1)^2} \le \left\| f_{\nu_q} - \left(f_{k_q} - \sum_{i=1}^{q-1} [Q_i - g_i] \right) \right\|_{L^\infty[0,1)^2} + \left\| \sum_{j=1}^{q-1} [Q_j - g_j] \right\|_{L^\infty[0,1)^2} + \\ + \|g_{\nu_q}^{(q+j_0)}\|_{L^\infty[0,1)^2} < 4^{-2q} + 4^{-q+2} + 2^{2q+8j_0} \|f_{\nu_q}(x)\|_{L^\infty[0,1)^2} < \frac{1}{\delta^2} 2^{-q+8}. \end{aligned} \tag{22}$$

Using induction one can find a sequence of functions $\{g_q(x,y)\}_{q=1}^\infty$, sets $\{E_q\}_{q=1}^\infty$, polynomials $\{Q_q(x,y)\}$, satisfying the conditions (16)–(22) for all $q \ge 1$. We put

$$E = \bigcap_{q=1}^{\infty} E_q. \tag{23}$$

From (5), (17), (22) and (23) it follows that

$$|E| > 1-\delta, \left\| \sum_{q=1}^{\infty} g_q \right\|_{L^\infty[0,1)^2} \leq \sum_{q=1}^{\infty} \|g_q\|_{L^\infty[0,1)^2} < \infty. \tag{24}$$

We define the function $\tilde{f}(x,y)$ and the sequence of numbers $\{\varepsilon_{k,s}\}_{k,s=0}^{\infty}$ in following way:

$$\tilde{f}(x,y) = \sum_{q=1}^{\infty} g_q(x,y), \tag{25}$$

$$\varepsilon_{k,s} = \begin{cases} \varepsilon_{k,s}^{(\nu_q,q+j_0)}; k,s \in \left[M_{\nu_q},^{(q+j_0-1)},\ M_{\nu_q}^{(q+j_0)}\right),\ q=1,2,\ldots, \\ 0;\ \ k,s \notin \bigcup\limits_{q=1}^{\infty} \left[M_{\nu_q}^{(q+j_0-1)},\ M_{\nu_q}^{(q+j_0)}\right). \end{cases} \tag{26}$$

From (12), (16), (18)–(26) it follows that

$$\tilde{f}(x,y) \in L^\infty[0,1]^2,\ \ \tilde{f}(x,y) = f(x,y),\ \ (x,y) \in E,$$

$$\left\| \sum_{k,s=0}^{M_{\nu_{q-1}}^{(q-1+j_0)}-1} \varepsilon_{k,s} b_{k,s} \varphi_k(x)\varphi_s(y) - \tilde{f} \right\|_\infty =$$

$$= \left\| \sum_{n=1}^{q-1} \left(\sum_{k=M_{\nu_n}^{(n+j_0-1)}}^{M_{\nu_n}^{(n+j_0)}-1} \varepsilon_{k,s}^{(\nu_n,n+j_0)} b_k \varphi_k(x)\varphi_s(y) \right) - \tilde{f} \right\|_\infty =$$

$$= \left\| \sum_{n=1}^{q-1} Q_n - \tilde{f} \right\|_{L^\infty[0,1)^2} \leq \left\| \sum_{n=1}^{q-1} (Q_n - g_n) \right\|_{L^\infty[0,1)^2} + \sum_{n=q}^{\infty} \|g_n\|_{L^\infty[0,1)^2} \leq 5\delta^{-2} 2^{-(q-10)}.$$

From this and (21) it follows that the series $\sum\limits_{k,s=0}^{\infty} \varepsilon_{k,s} b_{k,s} \varphi_k(x)\varphi_s(y)$ converges to the function $\tilde{f}(x,y)$ uniformly on $[0,1)^2$ and, therefore (see (11), (26)),

$$c_{k,s}(\tilde{f}) = \int_0^1 \int_0^1 \tilde{f}(x,y)\varphi_k(x)\varphi_s(y)dxdy = \varepsilon_{k,s} b_{k,s} = \varepsilon_{k,s} c_{k,s}(U),\ k,s = 0,1,2,\ldots$$

Theorem 2 is proved.

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ՈՒՈԼՇԻ ԿՐԿՆԱԿԻ ՀԱՄԱԿԱՐԳՈՎ ՖՈՒՐԻԵԻ ՀԱՎԱՍԱՐԱՉԱՓ ԶՈՒԳԱՄԻՏՈՒԹՅՈՒՆԸ

Աշխատանքում կառուցված է $U \in L^1[0,1)^2$ ունիվերսալ ֆունկցիա, որը Ուոլշի կրկնակի համակարգի նկատմամբ օժտված է ուղղման իմաստով ունիվերսալ հաըկությամբ:

М. Г. ГРИГОРЯН, А. Л. КАЗАРЯН, Г. Г. КАЗАРЯН

РАВНОМЕРНАЯ СХОДИМОСТЬ КОЭФФИЦИЕНТОВ ФУРЬЕ ПО ДВОЙНОЙ СИСТЕМЕ УОЛША

В работе построена универсальная функция $U \in L^1[0,1)^2$, которая по двойной системе Уолша обладает универсальным свойством в смысле модификации.