Modelling Home Ground Advantage (HGA) to Find the Probability to Win For Football Clubs Playing Against Each Other

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Մեփական դաշտի առավելության մոդելավորումը (HGA) միմյանց դեմ խաղացող ֆուտբոլային ակումբների հաղթանակի հավանականությունը գտնելու համար

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Ամփոփում: Սեփական դաշտի առավելությունը HGA (home ground advantage) հանդիսանում է ֆուտբոլային խաղի ելքի վրա ազդեցություն ունեցող ամենակարևոր հանգամանքներից մեկը: Որպեսզի ճիշտ գնահատել ֆուտբոլային խաղի ելքի հավանականությունը անպայման հարկավոր է հաշվի առնել փոփոխական HGA:
Փոփոխականը մոդելավորելով կարելի է տեղադրել մոդելի մեջ /օրինակ. the ordered probit model / , որպեսզի ավելի ճշգրիտ հնարավոր լինի կանխագուշակել խաղի ելքը: Տվյալ աշխատանքի մեջ քննարկվող մոդելը ունի հետևյալ տեսքը.
$y_{ij} = a_i - a_j + h_i + \varepsilon_{ij}$, որը ընդգրկում է թմերի բոլոր կարևոր հատկանիշները:
Վճռորոշ բառեր՝ սեփական դաշտի առավելությունը, ֆուտբոլային խաղի ելքի հավանականությունը, հաղթող տոկոսադրույքի մոդելավորում:

Моделирование преимущества своего поля (HGA) для нахождения вероятности победы футбольных клубов, играющих друг против друга

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Резюме: Преимущество своего поля HGA (home ground advantage) является одним из основных факторов влияющих на исход футбольного матча. Для того, чтобы правильно оценить вероятность исхода футбольного матча обязательно нужно рассматривать переменную HGA. Моделировав переменную можно поставить в модель (например, пробит–модель упорядоченного), чтобы более точно предсказать исход матча. Модель, рассматриваемая в данной работе имеет вид $y_{ij}=ai-aj+hi+\varepsilon_{ij}$, которая включает все важные характеристики команд.
Ключевые слова: преимущество своего поля, вероятность исхода футбольного матча, моделирование выигрышной маржи.

Introduction

Pollard (1986) has estimated the advantage of the team playing on its home ground in tournaments where each team plays an equal number of matches at home and away. He has concluded that the number of matches won by the teams playing at home, expressed in percentage from the total number of games, equals 50%. Although this method may be applicable to the medium of the whole tournament, it is apparently inadequate when studying the efficiency of individual clubs. The team playing home may win quite more or less than 50%, being a relatively strong or weak team. Snyderan Purdy (1985) showed the limitations of this approach when viewing the basketball university tournaments; they found out that the teams of Division 2 won only 40% of their home matches against the teams of Division 1. As the quality of teams differs we should consider the typical characteristic skills of the home and guest teams.

To estimate the advantages of the home field correctly we will need to model the team skills.

Modelling

The winning margin between the teams *i* and *j*, which play on the field of team *i* looks as follows:

$$y_{ij} = a_i - a_j + h_i + \varepsilon_{ij},$$

Where a_i and a_j - are the parameters of strength of teams *i* and *j* respectively, h_i - is the advantage of team *i* on its home ground, and ε_{ij} is the error. From the logical point of view the equation is quite simple. The winning margin equals the difference between the teams' strength, plus the advantage of the team which plays on its home ground, and the error. We assume that all a_i and h_i are constant values for the whole season. y_{ij}= (-1,0,1), depending on whether the host team won, the match ended in a draw or the guest team won respectively (such values of y_{ij} result from the ordered probit model). We may say that y_{ij} is the goal margin. For example, when the team wins at its home ground with the score 4:0, and as a guest team wins with the score 3:1, there is no advantage of home ground, if y_{ij} is not considered. But if it is, it shows a difference of 2 goals.

Using the Lagrange multiplier method to find the HGA

Let y_{ij} be the winning margin for the home team *i* against the team *j*. For N teams this produces a matrix of N×N with nulls on the major diagonal. Line folding gives a difference between the scored and missed goals of the guest team (HGD), and the folding by column gives a negative difference between the scored and the missed goals of the guest team (AGD).

For team i, $HGD_I = \sum_{j=1(j\neq I)}^{j=N} y_{Ij}$, $AGD_I = \sum_{i=1(i\neq I)}^{i=N} y_{iI}$

$$\sum_{i=1}^{i=N} HGD_I = -\sum_{i=1}^{i=N} AGD_I$$

As only differences a_i are used, and not relatives, we will limit to

$$\sum_{i=1}^{i=N} a_i = 0$$

Thus, we use the common Lagrange multiplier.

Minimization $S = \sum_{i=1}^{i=N}\sum_{j=1(j\neq i)}^{N}(y_{ij} - a_i + a_j - h_i)^2 + \lambda\sum_{i=1}^{i=N} a_i$

We differentiate by parts:

a_I: I = 1 to N, h_i: I = 1 to N, λ= 2N+1

(1) $-\sum_{j=1(j\neq I)}^{N} 2(y_{Ij} - a_I + a_j - h_I) + \sum_{i=1(i\neq I)}^{N} 2(y_{iI} - a_i + a_I - h_i) + \lambda = 0$, I=1 to N

(2) $-\sum_{j=1(j\neq I)}^{N} 2(y_{Ij} - a_I + a_j - h_I) = 0$, I=1 to N

$\sum_{i=1}^{i=N} a_i = 0$

Equation **(1)** results in:

$\sum_{j=1(j\neq I)}^{N} y_{Ij} = (N-1)a_I + (N-1)h_I - \sum_{j=1(j\neq I)}^{N} a_j$

$HGD_I = Na_I + (N-1)h_I - \sum_{j=1(j\neq I)}^{N} a_j$

(3) $HGD_I = Na_I + (N-1)h_I$

For I = 1 to N we have,

$\sum_{I=1}^{I=N} HGD_I = N\sum_{I=1}^{I=N} a_I + (N-1)\sum_{I=1}^{I=N} h_I$

(4) $HGD = (N-1)H$, where $H = \sum_{i=1}^{i=N} h_i$ total sum of home ground advantages of all individual teams

Supplying equation **(2)** into **(1)** we'll have,

$-\frac{\lambda}{2} = \sum_{i=1(i\neq I)}^{N}(y_{iI} - a_i + a_I - h_i) =$

$\sum_{i=1(i\neq I)}^{N} y_{iI} - \sum_{i=1(i\neq I)}^{N} a_i - \sum_{i=1(i\neq I)}^{N} h_i + (N-1)a_I =$

$= -AGD_I + a_I - H + h_I + (N-1)a_I$

(5) $-\frac{\lambda}{2} = -AGD_I - H + h_I + Na_I$

$-\sum_{I=1}^{I=N}\frac{\lambda}{2} = -\sum_{I=1}^{I=N} AGD_I - NH + \sum_{I=1}^{I=N} h_I + N\sum_{I=1}^{I=N} a_I$

$-N\frac{\lambda}{2} = HGD_I - (N-1)H + 0$

From **(4)** we have,

$$-N\frac{\lambda}{2} = HGD_I - HGD_I = 0 \Rightarrow \lambda = 0$$

As $\lambda = 0$, then **(5)** will turn into

(6) $AGD_I = -H + h_I + Na_I$

Supplying **(6)** into **(3)** we'll have,

$$HGD_I - AGD_I = Na_I + (N-1)h_I + H - h_I - Na_I$$

$$HGD_I - AGD_I = H + (N-2)h_I$$

By formula $HGD = (N-1)\,H$ we can calculate *H*, and knowing it, we can find individual values for h_I, using the formula $HGD_I - AGD_I = H + (N-2)h_I$.

The peculiarity of this procedure is that instead of complex regressive procedures we will use just a calculator to find h_I.

Table1

	Team	HW	HD	HL	Hf	Ha	HGD	AW	AD	AL	Af	Aa	AGD	h
1	Leicester	12	6	1	35	18	17	11	6	2	33	18	15	-0,20467
2	Manchester United	12	5	2	27	9	18	7	4	8	22	26	-4	1,22222
3	Arsenal	12	4	3	31	11	20	8	7	4	34	25	9	0,61111
4	Manchester City	12	2	5	47	21	26	7	7	5	24	20	4	1,22222
5	Tottenham	10	6	3	35	15	20	9	7	3	34	20	14	0,33333
6	Southampton	11	3	5	39	22	17	7	6	6	20	19	1	0,88889
7	WestHam	9	7	3	34	26	8	7	7	5	31	25	6	0,11111
8	Liverpool	8	8	3	33	22	11	8	4	7	30	28	2	0,5
9	Swansea	8	6	5	20	20	0	4	5	10	22	32	-10	0,55556
10	Newcastle	7	7	5	32	24	8	2	3	14	12	41	-29	2,05556
11	Stoke	8	4	7	22	24	-2	6	5	8	19	31	-12	0,55556
12	Sunderland	6	6	7	23	20	3	3	6	10	25	42	-17	1,11111
13	Chelsea	5	9	5	32	30	2	7	5	7	27	23	4	-0,11111
14	Watford	6	6	7	20	19	1	6	3	10	20	31	-11	0,66667
15	Everton	6	5	8	35	30	5	5	9	5	24	25	-1	0,33333
16	Norwich	6	5	8	26	30	-4	3	2	14	13	37	-24	1,11111
17	WestBrom	6	5	8	20	26	-6	4	8	7	14	22	-8	0,11111
18	CrystalPalace	6	3	10	19	23	-4	5	6	8	20	28	-8	0,22222
19	Bournemouth	5	5	9	23	34	-11	6	4	9	22	33	-11	0
20	AstonVilla	2	5	12	14	35	-21	1	3	15	13	41	-28	0,38889
														H= 5,684

We consider the EPL 2015-2016 season. The results of the season are known. There are *N* teams in the league, each of which plays $N-1$ times with other teams at home field, and $N-1$ times on the away field. Let us calculate h_I for all the clubs of the season. In Table 2 you can find the data for all the teams.

$H = \sum_{i=1}^{i=N} h_i = \sum_{i=1}^{i=N} HGD/\,(N-1)$, where H is the total sum of home ground advantages for all individual teams. H = 108/19 = 5,684.

For each team $h_I = (HGD_I - AGD_I - H)/(N-2)$, that is the home ground advantage for each team is the difference between the goals scored on the home field and those scored on the away field, subtracting H and divided by $(N-2)$. All the variables in the equation are known. For example, for *Southampton:*

$$h_{Sou} = \frac{17-1-5{,}684}{18} = 0{,}88889.$$

It is important to mention that there are teams which play on their home field worse than when away. In this case such a team is *Chelsea.*

$$h_{Che} = \frac{2 - 4 - 5{,}684}{18} = -0{,}11111$$

The $\ll - \gg$ sign shows that Chelsea has a negative advantage at home ground, which means the team plays worse at home than playing away. We assume that the scored and missed goals show more information than just wins, losses and draws. Table 1 confirms this easily. Playing on home field Chelsea has won 5 times, lost - 5 times and played draws for 9 times. Playing on the away field, the team has won 7 times, lost - 7 times and played draws for 5 times. So it turns out that Chelsea has scored 3*5 + 1*9 = 24 on the home ground, and 3*7 + 1*5 = 26 on the away ground. That is, the goals already contain information about losses and wins.

Conclusion

The present work makes it clear that to determine the match outcome there is no need to focus on the teams' strength and the position it has in the listing. There are a number of factors which impact the match outcome strongly as, for example, the home ground advantage is. After all there are teams which lose about once or twice in home matches per season, but at the same time win just thrice playing in the away matches. By using the HGA the probability of the match outcome may be predicted more accurately not only in football, but also in any other team sport.

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