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Պողոսյան Հայկ Ռուբիկի

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Ատենախոսության թեման հաստատվել է Ա. Ի. Ալիխանյանի անվան Ազգային գիտական լաբորատորիայի (ԱԱԳԼ) գիտական խորհրդում:

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Ֆ.մ.գ.դ. Շ. Բաբուջյան

Ֆ.մ.գ.թ. Վ. Պապոյան (ՄՀՄԻ, Դուբնա, ՌԴ)

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Abstract

The appearance of superstable cycles in the dynamical approach [1-2] to the antiferromagnetic and ferromagnetic spin-1 Ising and Ising-Heisenberg models [3-5] on diamond chains, and their connection with magnetization plateaus is investigated. The rational mappings, which provide the statistical properties of the model, are derived by using recurrence relations technique. We consider stability properties of the mapping, providing evidences of a connection between magnetization plateaus and dynamical properties, as the behavior of Lyapunov exponents [6-7]. The newfound correspondence between superstable point and zero magnetic plateau in spin-1 models on a diamond chain has been tested for a range of parameters.

The geometrically magnetic frustrations and quantum thermal entanglement of antiferromagnetic metal-containing compounds are considered on a diamond chain. We researched the magnetic and thermal properties of the symmetric Hubbard dimers with delocalized interstitial spins and the quantum entanglement states. It is presented magnetization plateaus and negativity in spin-1 Ising-Heisenberg model using transfer matrix technique [8].

The appearance of superstable points and cycles in the dynamical approach to the antiferromagnetic and ferromagnetic spin-1 Ising model on diamond-like decorated Bethe lattice, as well as their connection with magnetization plateaus are considered. The diamond chain can be considered as a coordination number equals to 2 diamond-like decorated Bethe lattice.

We investigate the interrelation between the distribution of stochastic fluctuations of independent random variables in probability theory and the distribution of time averages in deterministic Anosov C -systems. On the one hand, in probability theory, our interest dwells on three basic topics: the laws of large numbers, the central limit theorem and the law of the iterated logarithm for sequences of real-valued random variables. On the other hand, we have chaotic, uniformly hyperbolic Anosov C -systems [9-11] defined on tori which have mixing of all orders and nonzero Kolmogorov entropy. These extraordinary ergodic properties of Anosov C -systems ensure that the above classical limit theorems for sums of independent random variables in probability theory are fulfilled by the time averages for the sequences generated by the C -systems. The

MIXMAX generator of pseudorandom numbers represents the C-system for which the classical limit theorems are fulfilled.

We provide frontal cellular automata algorithms for Abelian sandpile models [12-13]. Also we calculate the height probabilities on a 2D lattice with fractal boundaries.

Timeliness and relevance

The theory of dynamical systems, besides its intrinsic theoretical relevance, recently had a significant impact on a wide range of disciplines from physics to ecology and economics, providing methodological tools which are currently employed in financial and economic forecasting, environmental modeling, medical diagnosis, industrial equipment diagnosis and so on. An important physical setting in which such dynamical techniques are profitably applied is equilibrium statistical mechanics of lattice models, more specifically in the investigation of physical properties of low-dimensional classic and quantum spin systems in an external magnetic field. Such systems can be successfully described with the help of Ising and Heisenberg models. The Ising model was invented by the physicist Wilhelm Lenz(1920), and solved by his student Ernst Ising. The one-dimensional model which was solved by Ising has no phase transition. The Ising model was further researched by Peierls, Onsager, Yang and many others. The model has also been successfully studied with the Monte Carlo method. The Heisenberg model was discovered by Werner Heisenberg and, nearly simultaneously, by P. A. M. Dirac, in 1926. Further work over the following decade established the Heisenberg model, due to its versatility, to be “the fundamental object of study of the theory of magnetism”. Since then, much work in theoretical statistical mechanics, solid state physics, and mathematical physics alike has been concentrated on understanding this model. Despite this effort, the Heisenberg model remains largely intractable, its predicted energy levels and eigenfunctions poorly understood, especially in a three-dimensional setting. The first step towards solving the Heisenberg model was the diagonalization by Hans Bethe in 1931 of the one-dimensional Heisenberg XXX model. Using an extension of the Bethe ansatz, Rodney Baxter solved the more general one-dimensional XYZ model in 1971. This solution, however, as well as approaches to the two-dimensional

and three-dimensional Heisenberg models, is exceedingly complicated. The Heisenberg model can be used to describe magnetically ordered solids, in which internal magnetic interactions cause individual magnetic ions to possess nonzero magnetic moments below some critical temperature. In papers [I-III] we study the spin-1 Ising and Ising-Heisenberg models on a diamond chain and diamond like decorated Bethe lattice.

Many numerical problems in science, engineering, finance, and statistics are solved nowadays through Monte Carlo methods; that is, through random experiments on a computer. There are two principal methods used to generate random numbers. The first method measures some physical phenomenon that is expected to be random and then compensates for possible biases in the measurement process. Example sources include measuring atmospheric noise, thermal noise, and other external electromagnetic and quantum phenomena. The second method uses computational algorithms that can produce long sequences of apparently random results, which are in fact completely determined by a shorter initial value, known as a seed value or key. While a pseudorandom number generator based solely on deterministic logic can never be regarded as a "true" random number source in the purest sense of the word, in practice they are generally sufficient even for demanding security-critical applications. From a set of sufficiently good generators one can get a superior one by mixing them which of course comes with a higher consumption of computation resources. This technique is a working solution for most cases. But with the emergence of tasks which require an astronomical large sets of random numbers the requirements for computational speed become crucial. The MIXMAX generator is a family of pseudorandom number generators (PRNG) and is based on Anosov C-systems (Anosov diffeomorphism) and Kolmogorov K-systems (Kolmogorov automorphism). To quantify the quality of a random generator is not a trivial task moreover when the generated sets are very large. In paper [IV] we studied the MIXMAX generator with the help of central limit theorems.

Cellular automata (henceforth: CA) are discrete, abstract computational systems that have proved useful both as general models of complexity and as more specific representations of non-linear dynamics in a variety of scientific fields. Despite functioning in a different way from traditional, Turing machine-like devices, CA with suitable rules can

emulate a universal Turing machine, and therefore compute, given Turing's thesis, anything computable.

The concept of Cellular Automata was originally discovered in the 1940s by John von Neumann. In the early Sixties, E.F. Moore (1962) and Myhill (1963) proved the Garden-of-Eden theorems stating conditions for the existence of so-called Gardens of Eden, i.e., patterns that cannot appear on the lattice of a CA except as initial conditions. Gustav Hedlund (1969) investigated cellular automata within the framework of symbolic dynamics. In 1977, Tommaso Toffoli used cellular automata to directly model physical laws, laying the foundations for the study of reversible CA (Toffoli 1977). The Abelian sandpile model, also known as the Bak–Tang–Wiesenfeld model, was the first discovered example of a dynamical system displaying self-organized criticality. It was introduced by Per Bak, Chao Tang and Kurt Wiesenfeld in a 1987 paper. The model is a cellular automaton. The model has since been studied on the infinite lattice, on other (non-square) lattices, and on arbitrary graphs (including directed multigraphs).

The original interest behind the model stemmed from the fact that in simulations on lattices, it is attracted to its critical state, at which point the correlation length of the system and the correlation time of the system go to infinity, without any fine tuning of a system parameter. This contrasts with earlier examples of critical phenomena, such as the phase transitions between solid and liquid, or liquid and gas, where the critical point can only be reached by precise tuning (e.g., of temperature). Hence, in the sandpile model we can say that the criticality is self-organized. The sandpile model exhibits a phenomenon called avalanches. The frequency, size, and duration of sandpile avalanches are of great interest. It has been demonstrated that the distribution of avalanches has a fractal structure, and thus has nontrivial correlations with power-law decay. Many phenomena in nature exhibit this fractal structure. In Dhar (1999), several examples of this power-law decay are explained. For example, the author cites spatial fractal behavior, such as the height profile of mountain ranges or the drainage area of a river as we travel downstream. In paper [V] we study the Abelian sandpile model with fractal boundaries using frontal cellular automata algorithms. One of the more interesting features of fractals is its way of scaling. If we take a polygon and double its edge length, its area becomes four times its original area. But if a one-dimensional

fractals lengths are all doubled, the spatial content of the fractal scales by a power that is not necessarily an integer. This power is called the fractal dimension of the fractal, and it usually exceeds the fractal's topological dimension. Fractal patterns which exhibit diverse degrees of self-similarity have been the focus of rendering or study in images, structures and sounds and have been observed in nature, technology, art, architecture and law. The graphs of most chaotic processes are fractal which makes them especially interesting to study from the perspective of Chaos theory.

Aim of the dissertation

- the study of stability of a number of classical and quantum spin systems on lattices,
- the calculation of Lyapunov spectrum for the mentioned models,
- the derivation of connection between Lyapunov exponents and magnetization,
- the derivation of connection between superstable point and the ground states phases,
- the study of Anosov-C systems as a RNG with Classical limit theorems,
- the study of Abelian sandpile model with fractal boundaries by utilizing frontal cellular automata algorithms.

Novelty of the work

In this work the following new results were obtained:

1. We found that superstable points correspond to a ground state phase (the none-magnetic phase) for some spin models.
2. Superstable points are connected to zero magnetization plateau for some spin models.

3. The first maximum Lyapunov exponent plateau is rather similar to the second plateau of magnetization if there exists a superstable point for some spin models.
4. The data suggests a non-trivial relation between ground states and stable fixed for some spin models.
5. We show that the random numbers generated with MIXMAX satisfy the Central limit theorems with satisfying accuracy.
6. We calculate the probability of having a fixed number of sand grains depending from the distance from the fractal boundary in ASM.

Practical value

The spin-1 Ising and Ising-Heisenberg models on diamond chains are a very good approximation for atoms of homometallic magnetic complex $[Ni_3(C_4H_2O_4)_2(\mu_3 - OH)_2(H_2O)_4]_n$, and the molecular compound $[Ni_8(\mu_3 - OH)_4(OMe)_2(O_3PR_1)_2(O_2C^tBu)_6(HO_2C^tBu)_8]$. Magnetic-property measurements on such compounds indicate the coexistence of both antiferromagnetic and ferromagnetic interactions between the magnetic centers, Ni ions with spin 1, which indeed suggests to investigate theoretically the magnetic properties of such compounds. Another related interesting material is $Cu_3(CO_3)_2(OH)_2$ -known as natural azurite (Copper Carbonate Hydroxide)- which can be well described by using the quantum antiferromagnetic Heisenberg model on a generalized diamond chain. The MIXMAX generator is used in a number of scientific programs, which makes the study of its quality actual. The Abelian sandpile model and fractals are used in a number of disciplines. The Abelian model can describe natural phenomenon such as the height profile of mountain ranges or the drainage area of a river as we travel downstream.

Main points to defend

Main points to defend are:

1. In spin-1 Ising/Ising-Heisenberg models on diamond chain and diamond like decorated Bethe lattice we found that superstable points which emerge when there is no external magnetic field correspond to a concrete ground state phase the non-magnetic phase. For the spin-1 Ising case on diamond chain the non-magnetic ground state phase corresponds to states with positive crystal field as for the Ising-Heisenberg case it corresponds to states with $\Delta > 2J$.
2. For the same models as mentioned in the first point we found that superstable points i.e. point whose maximum Lyapunov exponent tends to negative infinity when the temperature goes to zero and the slope connected to it correspond to zero magnetic plateaus.
3. For the models both maximum Lyapunov exponent and magnetization exhibit plateaus, which have similar divisions, except the superstable points zero magnetization correspondence there is also a connection between the first maximum Lyapunov exponent plateau and a magnetization plateau.
4. For the models mentioned above we found that the ground state diagrams and stable fixed points diagram are rather similar. If we did scale the stable fixed points diagram by a factor of 2 its divisions would have a one to one correspondence to the divisions of the ground state diagram.
5. We show that the random numbers generated with MIXMAX satisfy the Central limit theorems with satisfying accuracy. More precisely we generated arrays of random numbers (some with the length of 10^{14}) and tested if it satisfies the three basic topics: the laws of large numbers, the central limit theorem and the law of the iterated logarithm.
6. We calculate the height probabilities on a 2D lattice with fractal boundaries.

Length and structure of the dissertation

The thesis is divided into six parts. The first part is dedicated to some prerequisites on which we draw on later. The following five chapters describe our works, which are followed by the bibliography.

Content of the dissertation

The thesis starts with a short introduction. The first chapter explores a variety of topics such as the Ising model, Monte Carlo methods and Anosov C-systems, Cellular Automata and Sandpile model. In the next chapters we describe the original works.

Chapter 1

Chapter 1 begins with a short introduction to Statistical mechanics. The text covers the definitions of the partition function, free energy and the form of the observed average thermodynamic value of any observable in terms of Statistical mechanics. Next we give the definition of the general Ising model in form of its Hamiltonian, in this part only the most basic properties of the model are noted. In the following part we discuss specific heat, its form depending on internal energy which itself depends on free energy. In this part we also give the definition of two critical exponents. In the following part we give the definition of Magnetization as an average of magnetic moment per site and review some of its properties. This follows with the definition of susceptibility. After we consider the nearest-neighbor Ising model and discuss some of its interesting properties. Afterwards we discuss and review the one-dimensional Ising model. This part acts as a theoretical background for

some parts in Chapter 2, 3 and 4. We start this part by describing the model and giving the general form of its free energy and magnetization. We also discuss the transfer matrix method which has an essential role in exact solutions of Ising and/or Heisenberg models. Next we talk about the correlations between two spins. Here we also take advantage of the transfer matrix method. And at the closing part of this section we discuss the models critical behavior near zero temperature. In the following section we study the Ising model on Bethe Lattice. In the first part of the section we discuss the Bethe lattice and Cayley tree. We give a convenient definition of the Bethe lattice as a sub lattice of the Cayley tree. Next we show that the Bethe lattice has seemingly infinite dimensionality. In the next part we describe the recurrence relations technique. As the transfer matrix method it is also an essential tool in the study of Ising and Heisenberg models. Next we show the form of the partition function and magnetization when using the recurrence relations method. The set of recurrence relations expressions for the Ising model on Bethe lattice

has only one element and its form is comparatively simple. In the following part we take a closer look at the magnetization. We express it as a function of external magnetic field. We also discuss its behavior depending on the critical point. In the next part we take a look at the free energy of the Cayley tree and from it derived free energy of Bethe lattice.

In the next section we give an introduction to the Abelian sandpile model. This model was presented by Per Bak, Chao Tang and Kurt Wiesenfeld in a 1987 paper [1]. It is the first dynamical model shown to have self-organized criticality. Dynamical systems have self-organized criticality if they manifest a critical point as an attractor. Their macroscopic behavior thus displays the spatial and/or temporal scale-invariance characteristic of the critical point of a phase transition, but without the need to tune control parameters to a precise value, because the system, effectively, tunes itself as it evolves towards criticality. The concept self-organized criticality has been applied in a variety of disciplines including physical cosmology, evolutionary biology and ecology, bio-inspired computing and optimization (mathematics), economics, sociology, neurobiology and others. The model can be thought of as a Cellular automaton. The concept of cellular automata was first introduced by John von Neumann in the 1940s. In the following

part we give the definition of Cellular automata and discuss some of its more important properties. Next we review frontal cellular automata and Abelian sandpile model. We give the definition of frontal cellular automata and discuss its differences from the general cellular automata. We also bring arguments why frontal cellular automata are superior to general cellular automata in this setting. Next we give the definition of the general sandpile model. We also introduce a couple of concepts which simplify and enhance our understanding of the sandpile model. And at last but not least we give the criteria which makes a sandpile model Abelian and discuss some pivotal lemmas of the subject.

In the succeeding section we give an introduction to pseudo random number generators and especially the K-system generators. There are two methods of random number generation first the so called true random number generators and second pseudo random generators. True number generators first measure some physical phenomena then transform the output of the measurement to counter some possible biases. Of course the physical phenomena are chosen to be as random as possible. The most popular phenomena include atmospheric noise, thermal noise, and other external electromagnetic and quantum phenomena. One of the apparent weakness of such method is its speed. Most true random generators have much slower speed compared to pseudo random generators which makes them uncontentious if a large set of random numbers are required. Another shortcoming of true random generator is their unreliability, i.e. if we generate random numbers with the help of cosmic background radiance it will return different data depending on the radiance of the sun. The second group of random number generators are pseudo random generators or calls deterministic random bit generator. Pseudo random generators are essentially an algorithm which takes a seed and returns a sequence of numbers. Of course this sequence depends on the seed and is not truly random as the sequences generated by true random generators. Nevertheless, there are pseudo random generators which generate sequences very close too true random. An advantage of pseudo number generators is its reproducibility, i.e. by knowing the seed we can reproduce the sequence of numbers. Pseudo random generators are mostly used in simulations such as the Monte Carlo method and cryptography. Devising a pseudo number generator with quality output is not a trivial problem. Especially in cryptography and some Monte

Carlo simulations have some very high requirements on the pseudo random generator. In general, careful mathematical analysis is required to have any confidence that a pseudo random generator generates numbers that are sufficiently close to true random to suit the intended use. In the first part of this section we discuss the Monte Carlo method. Next we give the definition of the Metropolis algorithm. We also show the application of metropolis algorithm for the Ising model. After we give the definition of the so called K-system generators. Next we show some results connected to the K-system generators.

Chapter 2

Chapter 2 is based on the paper [I]. In this paper we consider the study the antiferromagnetic and ferromagnetic spin-1 Ising and Ising-Heisenberg models on diamond chains. The recursion relation technique is a powerful method which allows as to research the model as an abstract dynamical system. In this paper we investigate the connection between stability properties and magnetization. The paper is organized as follows:

Section 2.1 is an introduction.

Section 2.2 gives the definitions of the models we consider, and derives from them the recursion relation. It also gives the magnetization and quadrupole moments for the models.

Section 2.3 begins with the definition of Lyapunov exponents. Next it describes the process and results of our calculations. It also gives a comment on our results when compared with the experimental data from.

Section 2.4 gives discussion and conclusions.

In the Appendix we give the full form of the recursion relations and the eigenvalue, eigenvector pairs for the ground state phases.

Chapter 3

Chapter 3 is based on the paper [II]. In this paper the geometrically magnetic frustrations and quantum thermal entanglement of

antiferromagnetic metal-containing compounds are considered on a diamond chain. The paper is organized as follows:

Section 3.1 is an introduction.

Section 3.2 defines a symmetric diamond chain with delocalized Hubbard interstitial spins. The symmetric Ising-Hubbard model diamond chain is solved with the transfer matrix technique. This section also gives the expression for the reduced density matrix of a Hubbard dimer.

Section 3.3 describes the magnetization plateaus and negativity in spin-1 Ising - Heisenberg model. Which are also derived with the transfer matrix technique.

In Section 3.4 we utilize two models the spin-1 Ising and Ising-Heisenberg models. We calculate the magnetization and Lyapunov exponents using the recursion relation technique.

The last section gives a conclusions and final remarks.

Chapter 4

Chapter 4 is based on the paper [III]. In this paper we consider the antiferromagnetic and ferromagnetic spin-1 Ising on diamond-like decorated Bethe lattice. The calculations of Lyapunov exponents, magnetization and quadrupole moment are done via recursion relation technique. The paper is organized as follows:

Section 4.1 is an introduction.

Section 4.2 begins by giving the definition of the model and its dynamic solution in form of two recursion relations. Next it defines Lyapunov exponents and gives the conditions where superstable points occur. It also gives the ground states and magnetization of the model.

Section 4.2 gives discussion and concluding remarks on the paper.

Chapter 5

Chapter 5 is based on the paper [IV]. In this paper we investigate the interrelation between the distribution of stochastic fluctuations of independent random variables in probability theory and the distribution of time averages in deterministic Anosov C-systems. The paper is organized as follows:

Section 5.1 is an introduction.

Section 5.2 is a review of classical limit theorems in probability theory.

Section 5.3 discusses the statistical properties of deterministic dynamical C-systems.

Section 5.4 considers the MIXMAX C-systems Generator and its properties. It also studies the generator from the view point of central limit theorems.

Chapter 6

Chapter 6 is based on the paper [V]. The paper provides frontal cellular automata algorithms for abelian sandpile models. Also in this paper we calculate the height probabilities on a 2D lattice with fractal boundaries. Fractals have been subject of study in a variety of fields. The paper is organized as follows:

Section 6.1 is an introduction.

Section 6.2 is a short introduction to Cellular automata.

Section 6.3 begins with a brief introduction to Abelian sandpile model. Next it gives a frontal cellular automata algorithm for the model.

Section 6.4 is a short description of the loop erased random walk and Wilson's method.

Section 6.5 gives the results of the paper in form of the height probabilities on fractal bounded lattice.

Conclusions

We were able to derive the connection between Lyapunov exponents and magnetization for spin-1 Ising/Ising-Heisenberg models on diamond chain and diamond like decorated Bethe lattice. Namely the connection between super-stable point and the non-magnetic ground state phase. Also we observed the parallel between Lyapunov exponents plateaus and magnetization plateaus. Likewise, we noted a connection between ground state phases and stable fixed points.

We studied the MIXMAX generator from the point of view of classical limit theorems and found it fulfils the requirements.

We also studied the Abelian sandpile model on a two dimensional lattice with fractal boundaries using frontal cellular automata algorithms. Further we calculated its height probabilities.

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Կլասիկ և քվանտային սպինային համակարգերի
ուսումնասիրությունը ցանցերի վրա հաշվողական և
անալիտիկ մեթոդներով

Ամփոփում

Ուսումնասիրվել է սուպերկայուն ցիկլերի ի հայտ գալը դինամիկ մոտեցմբ հակֆերմագնիսական և ֆերմագնիսական սպին-1 Իզինգ և Իզինգ-Հայզենբերգ մոդելներում ադամանդի շղթայի վրա: Մոդելի վիճակագրական հատկությունները ապահովող ռացիոնալ արտապատկերումները դուրս են բերվել ռեկուրսիոն հարաբերությունների մեթոդի կիրառմամբ: Մենք դիտարկել ենք արտապատկերման կայունության հատկությունները, տրամադրելով ապացույցներ որոնք հաստատում են մագնիսացման հարթակների և դինամիկ հատկությունների միջև կապը: Համակարգի դինամիկ հատկությունները բնութագրել ենք Լյապունովի ցուցադրիչներ վարքի միջոցով: Ադամանդի շղթայի վրա սպին-1 մոդելներում սուպերկայուն կետի և զրոյական մագնիսական հարթակների միջև նոր հայտնագործված համապատասխանությունը փորձարկվել է մի շարք պարամետրերի համար:

Հակֆերմագնիսական մետաղ պարունակող միացությունների երկրաչափական մագնիսական խանգարումները և քվանտային ջերմային խճճվածությունը դիտարկվել են ադամանդի շղթայի վրա: Մենք ուսումնասիրել ենք դելոկալիզացված միջանցյալ սպիններով սիմետրիկ Հուբարդ դիմերների մագնիսական և

ջերմային հասկությունները և քվանտային խճճված վիճակները: Ներկայացվել է սպին-1 Բզինգ-Հայզենբերգ մոդելի մագնիսացման հարթակները և բացասականությանը օգտագործելով փոխանցման մատրից տեխնիկան:

Ուսումնասիրվել է սուպերկայուն ցիկլերի հայտնվելը դինամիկ մոտեցմբ հակֆերոմագնիսական և ֆերոմագնիսական սպին-1 Բզինգ մոդելում աղամանողով զարդարված Բեթեի ցանցի վրա: Աղամանողի շղթան կարելի է համարել աղամանողով զարդարված Բեթեի ցանց որի կոորդինացիոն համարը հավասար է երկուսի:

Մենք ուսումնասիրել ենք հավանականության տեսությունում անկախ պատահական փոփոխականների ստոխաստիկ տատանումների բաշխման և դետերմինիստիկ Անոտով C-համակարգերում ժամանակից կախված միջինների բաշխման միջև փոխհարաբերությունները: Մի կողմից, հավանականության տեսության մեջ, մեր հետաքրքրությունը կենտրոնացած է իրական արժեքներով պատահական փոփոխականների հաջորդականությունների համար մեծ թվերի օրենքի, կենտրոնական սահմանային թեորեմի և իտերացված լոգարիթմի օրենքի վրա: Մյուս կողմից, ունենք տորիի վրա սահմանված քառսային, համաչափ հիպերբոլիկ Անոտով C-համակարգեր որոնք ունեն բոլոր կարգերի խառնուրդներ և ոչ զրոյական Կոլմոգորովի էնտրոպիա: Անոտով C-համակարգերի արտակարգ էրգոտիկ հասկությունները ապահովում են վերը նշված հավանականության տեսության մեջ դասական սահմանային թեորեմների իրագործումը, որտեղ անկախ պատահական փոփոխականների գումարների դերը կատարվում են C-համակարգերի կողմից գեներացվող հաջորդականությունների համար ժամանակից կախված միջինները: Կեղծ պատահական թվերի MIXMAX- ի գեներատորը ներկայացնում է C-համակարգը որի համար կատարվում է դասական սահմանային թեորեմները: Մենք տրամադրել ենք ճակատային բջջային ավտոմատ տիպի ալերգիաները Աբելիան ավագակույտերի մոդելի համար: Նաև հաշվարկել ենք բարձրության հավանականությունները ֆրակտալային սահմաններով երկչափ ցանցի համար:

Погосян Айк Рубикович

**Вычислительные и аналитические исследования
классических и квантовых спиновых решетчатых
систем**

Резюме

Исследуется появление сверхстабильных циклов в динамическом подходе к антиферромагнитным и ферромагнитным моделям Изинга и Изинга-Гейзенберга на алмазных цепях и их связи с плато намагниченность. Рациональные отображения, обеспечивающие статистические свойства модели, выводятся с использованием техники рекуррентных соотношений. Рассматриваются свойства стабильности отображения, предоставляя доказательства связи между намагниченностью плато и динамическими свойствами, как поведение показателей Ляпунова. Было проверено новообетенная соответствие между сверхстабильной точкой и нулевым магнитным плато в моделях спина-1 на алмазной цепи для ряда параметров. Рассматриваются геометрически магнитные фрустрации и квантовое тепловое запутанность антиферромагнитных металлсодержащих соединений на алмазной цепочке. Мы исследовали магнитные и термические свойства симметричных димеров Хаббарда с делокализованными междуузельными спинами и состояниями

квантовых запутываний. Представлены намагничивающие плато и отрицательность в спин-1 модели Изинга-Гейзенберга с использованием метода трансферной матрицы.

Рассмотрено появление суперстабильных точек и циклов в динамическом подходе антиферромагнитных и ферромагнитных моделей со спином 1 на алмазоподобной решетке Бете. А также рассмотрена связь взаимодействий с плато намагниченности. Алмазную цепь можно рассматривать как алмазоподобный решетка Бете у которого координационное число, равное двум.

Исследуется взаимосвязь между распределением стохастических флуктуаций независимых случайных величин в теории вероятностей и распределением средних по времени в детерминированных C -системах Аносова. С одной стороны, в теории вероятностей наш интерес затрагивает три основные темы: законы больших чисел, центральную предельную теорему и закон повторного логарифма для последовательностей вещественных случайных величин. С другой стороны, мы имеем хаотичные, равномерно гиперболические C -системы Аносова, определенные на торах, у которого есть смешивание всех порядков порядка и отличную от нуля энтропию Колмогорова. Экстраординарные эргодические свойства C -систем Аносова гарантируют, что приведенные выше классические предельные теоремы для сумм независимых случайных величин в теории вероятностей выполняются средними времени для последовательностей, порождаемых C -системами. Генератор псевдослучайных чисел MIXMAX представляет собой C -систему, для которой выполняются классические предельные теоремы.

Мы предоставляем алгоритмы лобовых клеточных автоматов для моделей абелевых песчаной кучи. Также мы вычисляем вероятности высоты на двумерной решетке с границами фракталов.